Supplemental Material

Mathematical Model

Here we develop a model to capture the dune subsidence over time in our experiments. We assume that the polymer is not compressible and thus it maintains a constant density through time. To determine the dune subsidence, we must quantify the displacement of polymer from beneath the dune as it flows across the flume. Consider a cross-sectional view of a dune on a salt layer of uniform thickness (Figure 2). A triangular-shaped dune subsides into the underlying salt layer without changes in the initial width of the deposit, for simplicity. Changes in salt volume underneath of the dune occur by outflow of the salt from beneath the dune.

\[
\frac{d(\eta_s B_d - 0.5 w B_d)}{dt} = q_s \tag{S1}
\]

where \(B_d\) is the width of the dune, \(\eta_s\) is the initial thickness of the salt layer, \(w\) is the vertical deflection in the center of the loaded region, and \(q_s\) is the volumetric discharge per unit width of the salt outward from beneath the dune.

An equation for the flux \((q_s)\) is required to solve (S1). We apply the solution for viscous flow between two rigid plates. The no-slip condition is applied to the boundaries between the salt layer and the plates. The velocity profile is a parabola:

\[
U(y) = \frac{1}{2 \mu} \frac{dp}{dx} (y^2 - by) \tag{S2}
\]

where \(y\) is the distance from the front plate to a point within the viscous layer, \(dp/dx\) is the pressure gradient pushing the salt flow, \(b\) is the width of the viscous layer, and \(\mu\) is the viscosity of the polymer.

The velocity can also be described in the vertical direction by the solution for viscous flow between a rigid surface and a free surface:

\[
U(z) = -\frac{1}{2 \mu} \frac{dp}{dx} (2 \eta_s z - z^3) \tag{S3}
\]

where \(z\) is the distance from the bottom rigid surface to a point within the viscous layer. The salt flux can be determined by integrating the velocity profiles:

\[
q_s = \int_0^h U(y) dy = \frac{1}{18 \mu} \frac{dp}{dx} h^3 \tag{S5}
\]

The pressure gradient that drives salt motion arises from the difference in the loading of sediment by the dune’s geometry. As the salt is displaced from beneath the dune, it accumulates against the flume edges as something like salt diapirs that counteract outflow of the salt from beneath of the dune. The mean pressure in the salt layer can be defined as \(\rho_s g \eta_s/2\), and the
pressure gradient therefore takes the following form:

\[
\frac{dp}{dx} = \left( \frac{\rho_s g \eta_d + \rho_s g (\eta_x - w) - \rho_s g \eta_x}{2B_d} \right) + \left( \frac{\rho_s g (\eta_x + v) - \rho_s g \eta_x}{L - B_d} \right) \tag{S6}
\]

where \( v \) denotes the upwelling height of the salt which can be approximated as

\[
v = \frac{B_d}{L - B_d} w \tag{S7}
\]

Combining (S6) and (S7) with (S5), and insertion of the flux equation (S5) into the volume conservation (S1) provides

\[
\frac{d}{dt} (B_d w) = \frac{1}{9\mu 2B_d} B^3 \left( \rho_s g \eta_d - \rho_s g w \left[ 1 + 2 \left( \frac{B_d}{L - B_d} \right)^2 \right] \right). \tag{S8}
\]

Assuming the width of the dune is static, (S8) can be further simplified to solve for the rate of dune subsidence into the deformable salt substrate:

\[
\frac{dw}{dt} = \Phi_d \eta_d - \Phi_s w \tag{S9}
\]

where

\[
\Phi_d = \frac{1}{9\mu 2B_d} \rho_s g B^3 \tag{S10a}
\]
\[
\Phi_s = \frac{1}{9\mu 2B_d} \rho_s g \left[ 1 + 2 \left( \frac{B_d}{L - B_d} \right)^2 \right] \tag{S10b}
\]

For the case of the natural system, we begin with the same relation for salt flux (S1) but assume an infinite basin width \( b \), such that this width is not a control on the amount or rate of subsidence. Instead, we use the parabolic vertical profile of outflow velocity to calculate viscous flow between a horizontal plate and a free surface, in which velocity is minimum at the basement and maximum at the free surface.

\[
U(z) = -\frac{1}{2\mu} \frac{dp}{dx} \left( 2\eta_x z^2 \right) \tag{S11}
\]

where \( z \) is the upward distance from the bottom boundary. As shown in (S5), we can integrate (S11) to find an equation for the outflow of salt from beneath the dune and equate it to the previous relation for flux \( q_s \) (S1):

\[
\frac{d}{dt} \left( \eta_x B_d - 0.5wB_d \right) = q_s = -\frac{1}{3\mu} \frac{dp}{dx} \eta_x^3
\]
Applying the same pressure gradient acting across the dune and interdune surfaces (S6), we arrive at the same formula as in (S9) but different variables for $\Phi_s$ and $\Phi_d$:

\[
\frac{dw}{dt} = \Phi_d \eta_d - \Phi_s w
\]  
(S12)

\[
\Phi_d = \frac{1}{3 \mu} \frac{\rho_d g}{B_d^2} \eta_s^3
\]  
(S13a)

\[
\Phi_s = \frac{1}{3 \mu} \frac{\rho_s g}{B_d^2} \eta_s^3 \left[1 + \left(\frac{B_d}{L - B_d}\right)^2\right]
\]  
(S13b)

Table S1: Parameter values and descriptions for the physical experiments and for the natural system (values used in the mathematical model).

<table>
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<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
<th>Description</th>
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<td>Width of dune and two interdunes</td>
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