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Spatial autoregressive structure in meander evolution: Appendixes

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APPENDIXES

1 Average Migration Rate

Equations (1) and (2) in the text can be written

$$
\zeta = -E_0 \chi \int_{-\infty}^{\infty} \delta(s - s') C^*(s') \, ds' + E_0 C_f (A + 2) \chi^2 \int_{0}^{\infty} e^{-2x C_f s'} C^*(s - s') \, ds'
$$

The term containing the delta function can be reduced (Hochstadt, 1964, p.165) such that

$$
\zeta = -E_0 \chi C^*(s) + E_0 C_f (A + 2) \chi^2 \int_{0}^{\infty} e^{-2x C_f s'} C^*(s - s') \, ds' \quad (1.1)
$$

Setting $C^*$ to a constant value $K$, (1.1) has the solution

$$
\zeta(\infty) = E_0 A \chi K/2 \quad (1.2)
$$

The quantity $E_0 A \chi/2$ in (1.2) may be interpreted as a steady state gain $g_s$. The gain of a system in general equals its output value, in this case $\zeta$, eventually reached in response to a unit step input, in this case $C^*$ (Figure 1.1). Parker and Andrews (1986) use (1.2) to estimate a value of $E_0$ for the Beatton River, Canada. Relevant data come from the works of Hickin and Nanson (1975), Nanson (1977) and Nanson and Hickin (1983), which concern the relation between migration rate and bend curvature (Figure 4) based on isochrones of point bar surfaces. Estimating a value of $E_0$ is reexamined here.

As Howard and Knutson (1984) and Parker and Andrews (1986) note, the form of (1.2) pertains to the abstraction of an infinitely long bend with constant curvature. However, the data of Hickin and Nanson (Figure 4) involve bends with finite lengths, each with varying curvature between crossovers. Moreover, these migration rates and bend curvatures are averaged.
simultaneously over space (individual meander scrolls) and over time (successive scrolls). Therefore, it is useful to consider a spatially-averaged version of (1.1). An appropriate form is:

\[ \overline{\zeta} = g_s \overline{C^*} + a \]  

(1.3)

where bars denote spatial averages for an individual scroll, and \( a \) is a residual.

To recast (1.1) into the form of (1.3), consider a bend that is preceded by a long straight reach. Let \( \hat{l} \) denote the arc distance in a downstream direction with origin at the bend entrance, and let \( \hat{S} \) denote the arc length over which an average migration rate and an average curvature are to be estimated. Dimensionless variables then are defined

\[ l = \hat{l}/H_0; \quad S = \hat{S}/H_0 \]

Also, define a deviation in curvature \( c^* \) such that

\[ C^* = \overline{C^*} + c^*(s) \]  

(1.4)

This expression (1.4) of local curvature \( C^* \) in terms of a mean bend value \( \overline{C^*} \) and a local deviation \( c^* \) can be substituted directly into (1.1):

\[ \zeta = -E_0 \chi [\overline{C^*} + c^*(s)] + E_0 C_f (A + 2) \chi^2 \int_0^\varepsilon e^{-2x_{C_f}} [\overline{C^*} + c^*(s - s')] ds' \]

Expanding and collecting terms in \( \overline{C^*} \) and \( c^* \)

\[ \zeta = -E_0 \chi \overline{C^*} + E_0 C_f (A + 2) \chi^2 \overline{C^*} \int_0^\varepsilon e^{-2x_{C_f}} ds' \]

\[ - E_0 \chi c^*(s) + E_0 C_f (A + 2) \chi^2 \int_0^\varepsilon e^{-2x_{C_f}} c^*(s - s') ds' \]  

(1.5)

At an arc distance \( l \) from the entrance of a bend that is preceded by a long straight reach, the upper limit of integration in (1.5) becomes \( l \), and (1.5) reduces to
\[
\zeta = -E_0 x C^* + \frac{E_0 C_f x^2 (A + 2)}{2C_f} (1 - e^{2xC_f}) C^*
\]

\[- E_0 x^2 c^*(l) + E_0 C_f (A + 2)x^2 \int_0^{l} e^{-2xC_f} c^*(s - s') ds' \]  

(1.6)

Now, the average migration rate over a bend of arc length \( S \) is

\[
\bar{\zeta} = \frac{1}{S} \int_0^{S} \zeta(l) dl
\]

Substituting the expression (1.6) for \( \zeta(l) \) then leads to

\[
\bar{\zeta} = E_0 [-x + \frac{(A + 2)x}{2} + A + 2 \int_0^{S} e^{-2xC_f} - 1] C^*
\]

\[- E_0 x \int_0^{S} c^*(l) dl + E_0 C_f (A + 2) \int_0^{S} \int_0^{l} e^{-2xC_f} c^*(s - s') ds' ds' \]  

(1.7)

The term containing the single integral equals zero, following the definition of (1.4). Equation (1.7) thus has the form of (1.3), where the residual term consists of a mean, weighted aggregate of curvature deviations \( c^* \) from the bend average \( \overline{C^*} \). In general, this residual should be small if deviations are small relative to the average; influences of small positive and negative curvature deviations tend to be canceling. Numerical analyses involving curvature data from bends one, four, nine and 10 of Hickin and Nanson (1975) indicate that the last term in (1.7) contributes only one or two percent to the average rate. These bends were selected because values of the length \( S \) can be reasonably estimated: upstream ends of scroll bars are mostly preserved, not truncated by downvalley migration of upstream bends. Moreover, the approximate joining of isochrones at each bend entrance indicates that little migration has occurred there. These conditions are compatible with the assumptions leading to (1.7) (Furbish, 1988).

From the text, \( S = \hat{S}/H_0 \), and the central angle \( \theta' = b\theta/H_0 \) so that \( \theta = \theta' H_0/b \). Now, define two additional dimensionless variables consistent with those first introduced in the text:

\[
\hat{S} = \theta \overline{r_0}; \quad \overline{C^*} = b/\overline{r_0}
\]

Thus, \( r_0 = b/C^* \), whence substitutions for \( \hat{S}, \overline{r_0} \) and \( \theta \) lead to \( S = \theta'/C^* \). Substituting this for
in Equation (1.7), and neglecting its last two terms, leads to

\[
\bar{\xi} = E \bar{C} [-\chi + \frac{A}{2} (A + 2) + \frac{A + 2}{4C\theta'} (e^{-2\bar{C}^2\theta'/\bar{C}^2} - 1)\bar{C}^2]\bar{C}^2
\]

which is Equation (3) in the text. This casts (1.7) in terms of the angle \(\theta^*\), a parameter that is easy to measure.

Curvatures measured by Hickin and Nanson (Figure 4) have a range of 0.04 to nearly 0.4. In contrast, curvatures estimated here as spatial averages of individual isochrones (Figure 3) have a range of about 0.05 to 0.2. This difference is attributable to the procedures used to estimate average curvature. Nanson and Hickin (1983) estimate \(\bar{C}^*\) as the arithmetic mean of two curvatures derived from circular arcs, one associated with the curvature apex and one associated with the reaches upstream and downstream from the apex. This procedure overweights the curvature apex since the apex region generally is only a small portion of the total bend. The procedure therefore overestimates average curvature, even when adjusted to reflect a centerline value. Consequently, values of \(\bar{C}^*\) estimated here are probably less than values estimated by Nanson and Hickin (1983), and the domain of \(\bar{C}^*\) covered by data in Figure 3 includes most of the domain covered in Figure 4 (Furbish, 1988).

In addition, the Hickin-Nanson data (Figure 4) suggest that a maximum migration rate occurs at an intermediate curvature value. However, such a maximum is not readily apparent in the spatially-averaged data (Figure 3). For simple geometrical reasons, large curvatures typically are not sustained over the full lengths of large bends (Figure 1.2), and sharply curved bends typically are short bends (e.g., bends three, eight and 10, Figure 7 and 9 of Hickin and Nanson (1975)). It therefore may be unusual for a bend to have a sufficient length of large curvature that the near-bank velocity reaches a maximum possible asymptotic value, in the sense of relation (1.2) (Figure 1.1), commensurate with this local, large curvature. In such cases, a limited opportunity exists for observing high erosion rates in association with high average curvatures.

2 Discrete Migration Process

Consider the possibility that centerline curvature \(C^*\) consists of a series of piecewise continuous square waves (pulsed input). This coincides with the abstraction of a series of circular-arc bend segments, each segment representing an individual pulse in the curvature series. Linearity of the problem ensures that the continuous output -- the local migration rate \(\xi\) -- consists of the superposition of individual responses to successive pulses (Figure 2.1). With equispaced pulses, and from (1.1), the output at position \(s\) is given exactly by
\[ \zeta(s) = -E_0xC^*(s) + E_0C_f(A + 2)\chi^2 \left( \int_0^1 e^{-2xC_f's'} ds' \right) C^*_{s-1}. \]

\[ + \left( \int_1^2 e^{-2xC_f's'} ds' \right) C^*_{s-2.} + \left( \int_2^3 e^{-2xC_f's'} ds' \right) C^*_{s-3.} + \ldots \] (2.1)

where the notation indicates that the curvature \( C^* \) is constant over each interval \( s - i \) to \( s - i + 1 \). Independently of the values of curvature \( C^*_{s-i} \), each of the bracketed integrals takes on a specific, finite value such that (2.1) may be written as a discrete process

\[ \zeta(s) = \zeta_s = kC^*_{s-1} + k_0C^*_{s-2} + k_1C^*_{s-3} + k_2C^*_{s-4} + \ldots \] (2.2)

where \( k = E_0\chi \) and each of the coefficients

\[ k_i = E_0C_f(A + 2)\chi^2 \int_1^{i+1} e^{-2xC_f's'} ds' \]

The set of coefficients \( k \) plus \( k_i \) constitutes a discrete impulse response function, parallel to the continuous impulse response function (2). Thus, a discrete response function exists such that, for a pulsed input, the continuous output \( \xi(s) \) equals the discrete output \( \zeta_s \) at positions \( s, s-1, s-2, \ldots \) (e.g. Box and Jenkins, 1976; p. 356-357).

A smooth curvature series can be approximated as a series of pulses (Figure 2.2). Now, in contrast to the previous result for a pulsed input, a set of coefficients \( k \) plus \( k_i \) generally does not exist such that \( \xi(s) = \zeta_s \) for \( s, s-1, s-2, \ldots \) Nonetheless, the approximation (compare with 2.2)

\[ \zeta(s) = \zeta_s = kC^*_{s} + k_0C^*_{s-1} + k_1C^*_{s-2} + k_2C^*_{s-3} + \ldots + N_s \] (2.3)

can be made arbitrarily close by selecting a sufficiently small space increment \( \Delta s \). \( N_s \) denotes error of the approximation and any other noise manifest at position \( s \). This can be written in the more compact form of Equation (4) in the text:

\[ \zeta_s = -kC^*_{s} + k^i \sum_{j=0}^{\infty} \phi^j C^*_{s-j} \]

where
\[ k = E_0 \chi; \quad k' = \frac{E_0 \chi}{2} (A + 2)(1 - \phi'); \quad \phi' = e^{-2\chi C \Delta s^j} \]

Since the weighting function \( \phi' \) is independent of the curvature series \( \{C^*_s\} \), (2.3) can be expressed as the finite autoregressive process

\[ \zeta_s + kC^*_s = \phi(\zeta_{s-1} + kC^*_{s-1}) + k'C^*_s \tag{2.4} \]

which, when rearranged to

\[ \zeta_s + (k - k')C^*_s = \phi(\zeta_{s-1} + kC^*_{s-1}) \]

is equation (6) in the text with \( j = 1 \). To show the equivalence of (2.4) to the infinite series (2.3), it is sufficient to perform a recursive substitution. From (2.4)

\[ \zeta_{s-1} + kC^*_{s-1} = \phi(\zeta_{s-2} + kC^*_{s-2}) + k'C^*_{s-1}, \]

\[ \zeta_{s-2} + kC^*_{s-2} = \phi(\zeta_{s-3} + kC^*_{s-3}) + k'C^*_{s-2} \]

and so forth for \( \zeta_{s-3}, \zeta_{s-4}, \ldots \). Substituting once into (2.4)

\[ \zeta_s + kC^*_s = \phi[\phi(\zeta_{s-2} + kC^*_{s-2}) + k'C^*_{s-1}] + k'C^*_s \]

\[ = \phi k'C^*_s + \phi^2(\zeta_{s-2} + kC^*_{s-2}) + k'C^*_s \]

Substituting a second time

\[ \zeta_s + kC^*_s = \phi k'C^*_s + \phi^2[\phi(\zeta_{s-3} + kC^*_{s-3}) + k'C^*_{s-2}] + k'C^*_s \]

\[ = \phi k'C^*_s + \phi^2 k'C^*_{s-2} + \phi^3(\zeta_{s-3} + kC^*_{s-3}) + k'C^*_s \]

and so forth for terms in \( \zeta_{s-3}, \zeta_{s-4}, \ldots \). Thus, indefinite substitution retrieves the infinite series (2.3).
3 Flow History

As with spatially random features, there is a need to consider effects of arbitrary time series of flows, which are essentially independent of channel characteristics. (This is not entirely true insofar as channel characteristics modulate the movement of a flood wave through it.) But typically only a brief part of the flow history of a given stream is recorded. In view of this uncertainty, we can at best describe the response to a flow series in a probabilistic sense. Here also the autocorrelation structure of the flow series is important. Descriptions of this structure have traditionally relied on simple "parameters" of the flow history based on return periods (e.g., mean annual flow) or on morphologic criteria (e.g., bankfull flow) to correlate with channel geometry. These measures ignore the importance of serial dependence in flows and in channel response. This inadequacy was initially addressed by Pickup and Rieger (1979) and more recently by Yu and Wolman (1987), who conceptualized channel response in terms of a temporal convolution. (Curiously, we seem to be more comfortable with describing flow series as random functions than with treating channel features using similar mathematics.) In addition, Seminara and Tubino (1989) are now considering effects of unsteady flow on bar growth.

4 Erodibility as a Random Function

The erodibility coefficient $E_0$ can be factored from (8) in the text and treated as a random function of $s$, such that (8) takes the form

$$\zeta_s = \frac{\omega'(B)}{\delta(B)} E_s C_s' + N_s$$  \hspace{1cm} (4.1)

where the prime denotes that $E_0$ has been factored out of the parameters $\omega$, and $E = E_0$; the subscript "zero" has been dropped for simplicity. Now, the erodibility series $\{E_s\}$ at any given instant can be represented by a difference equation having the form of (7). This is symbolized herein by

$$E_s = \overline{E_s} + \frac{\eta(B)}{\alpha(B)} a'_s$$  \hspace{1cm} (4.2)

where $\overline{E_s}$ is the mean erodibility, and $a'_s$ is assumed to be a purely random (white) noise in the sense that neighboring values are uncorrelated.

Consider the possibility that erodibility $E_s$ varies along a channel. Substituting (4.2) into (4.1) and expanding...
\[ \zeta_s = \frac{\omega'(B)}{\delta(B)} C_s^* E_s + \frac{\omega'(B)}{\delta(B)} C_s^* \frac{\eta(B)}{\alpha(B)} a_s' + N_s \]  

(4.3)

The term containing \( E_s \) may be considered a "primary" influence on the migration rate \( \zeta_s \) deriving from the convolutional structure described above. It coincides with the migration rate expected under the condition where erodibility is spatially constant and equal to the mean erodibility. The term containing \( a_s' \), in the general case, leads to serially correlated deviations in \( \zeta_s \) about the primary level. Both the magnitude and the variability of these deviations are modulated by the quantity \( \omega'(B)C^*/\delta(B) \), which equals the magnitude of the near-bank streamwise flow velocity. In the special case that erodibility varies purely randomly, (4.2) reduces to \( E_s = E_s + a_s' \), and the term in (4.3) containing \( a_s' \) reduces to \( \omega'(B)C^*a_s'/\delta(B) \). This is serially uncorrelated noise whose variability is proportional to the near-bank velocity.

Thus, in general there is to be expected a greater variability in migration rates at positions where the rates tend to be large versus where they tend to be small. In addition, variations in migration rates about the primary level will depend on how \( E_s \) changes along the channel — that is on the form of (4.2). Little is actually known about this form; I am unaware of any cases where variations in erodibility have been systematically mapped along a channel, or inferred from the areal distributions of different sediments and vegetation over a valley floor. Certainly, such variations contribute to diversity of bend-form shapes as bends locally encounter relatively tough, or erodible, material during migration. However, erodibility is assumed to be constant in the text, which physically represents the case of homogeneous valley-floor sediments. This allows for the effects of an irregular curvature series on train evolution to be examined independently of effects due to varying erodibility.

5 Gain and Phase Functions

The frequency response function \( F(f) \) of a linear process having the form of (7) in the text is (e.g. Box and Jenkins, 1976)

\[ F(f) = \frac{\omega_0 - \omega_1 e^{-12\pi f} - \ldots - \omega_u e^{-12\pi f}}{1 - \delta_1 e^{-12\pi f} - \ldots - \delta_v e^{-12\pi f}} ; \quad 0 \leq f \leq \frac{1}{2} \]  

(5.1)

and its squared gain function is

\[ G^2(f) = \frac{|\omega_0 - \omega_1 e^{-12\pi f} - \ldots - \omega_u e^{-12\pi f}|^2}{|1 - \delta_1 e^{-12\pi f} - \ldots - \delta_v e^{-12\pi f}|^2} ; \quad 0 \leq f \leq \frac{1}{2} \]  

(5.2)

where \( i \) is the imaginary number \( \sqrt{-1} \). Bars denote the magnitude of a complex number \( W = (W_1, W_2) \); that is \( |W| = \sqrt{W_1^2 + W_2^2} \), where \( W_1 \) is the real part and \( W_2 \) is the imaginary part.
of \( W \). With moving-average and autoregressive orders \( u = v = 1 \), and using the Euler relation \( \exp(-i\psi) = \cos\psi - i\sin\psi \), (5.2) becomes

\[
G_1^2(f) = \frac{(\omega_0 - \omega_1\cos2\pi f)^2 + (i\omega_1\sin2\pi f)^2}{(1 - \delta_1\cos2\pi f)^2 + (i\delta_1\sin2\pi f)^2}; \quad 0 \leq f \leq \frac{1}{2}
\]

After squaring terms, and using the identity \( \cos^2\psi + \sin^2\psi = 1 \), this becomes Equation (11) in the text

\[
G_1^2(f) = \frac{\omega_0^2 + \omega_1^2 - 2\omega_0\omega_1\cos2\pi f}{1 + \delta_1^2 - 2\delta_1\cos2\pi f}; \quad 0 \leq f \leq \frac{1}{2} \tag{5.3}
\]

Treating curvature \( C^* \) as a second-order autoregressive process in Equation (9), such that \( \varepsilon_0 = 1 \) and \( v = 2 \), a second squared gain function \( G_2^2(f) \) is similarly derived from an equation having the form of (5.2), but with coefficients symbolized as in (9):

\[
G_2^2(f) = \frac{1}{1 + \gamma_1^2 + \gamma_2^2 - 2\gamma_1(1 - \gamma_2)\cos2\pi f - 2\gamma_2\cos4\pi f}; \quad 0 \leq f \leq \frac{1}{2} \tag{5.4}
\]

The overall gain \( G(f) \) of systems in series -- where the output of one system is the input for the next -- equals the product of the individual gains. Thus, \( G(f) = G_1(f)G_2(f) \) implying that \( G^2(f) = G_1^2(f)G_2^2(f) \), and from (5.3) and (5.4)

\[
G^2(f) = \frac{(\omega_0^2 + \omega_1^2 - 2\omega_0\omega_1\cos2\pi f)}{\Lambda}; \quad 0 \leq f \leq \frac{1}{2}
\]

where \( \Lambda \) is

\[
1 + \delta_1^2 + \gamma_1^2 + \gamma_2^2 + 2\delta_1\gamma_1 + \delta_1^2\gamma_1^2 + \delta_1^2\gamma_2^2 - 2\delta_1\gamma_1\gamma_2 + (-2\delta_1 - 2\gamma_1 + 2\delta_1\gamma_2 + 2\gamma_1\gamma_2 - 2\delta_1^2\gamma_1^2 - 2\delta_1\gamma_1\gamma_2^2 - 2\delta_1^2\gamma_1\gamma_2)\cos2\pi f
\]

\[
+ (-2\gamma_2 + 2\delta_1\gamma_1 - 2\delta_1^2\gamma_2 - 2\delta_1\gamma_1\gamma_2)\cos4\pi f + 2\delta_1\gamma_2\cos6\pi f
\]

which is Equation (13) in the text.
The phase function of a linear process having the form of (7) is found by expanding (5.1) using the Euler relation given above, then collecting real and imaginary parts. With \( u = v = 1 \),

\[
F(f) = \frac{\omega_0 - \omega_1 \cos 2\pi f + i\omega_1 \sin 2\pi f}{1 - \delta_1 \cos 2\pi f + i\delta_1 \sin 2\pi f}; \quad 0 \leq f \leq \frac{1}{2}
\]

Dividing (e.g. LePage, 1980, p. 20), this can be written as the sum of real and imaginary parts

\[
F(f) = W_1 + iW_2
\]

\[
= \frac{(\omega_0 - \omega_1 \cos 2\pi f)(1 - \delta_1 \cos 2\pi f) + \omega_1 \delta_1 \sin 2\pi f}{(1 - \delta_1 \cos 2\pi f)^2 + (\delta_1 \sin 2\pi f)^2}
\]

\[
+ i\frac{\omega_1 \sin 2\pi f(1 - \delta_1 \cos 2\pi f) - \delta_1 \sin 2\pi f(\omega_0 - \omega_1 \cos 2\pi f)}{(1 - \delta_1 \cos 2\pi f)^2 + (\delta_1 \sin 2\pi f)^2}; \quad 0 \leq f \leq \frac{1}{2}
\]

The phase function is given by \( P(f) = \arctan(-W_2/W_1) \), whence algebra leads to

\[
P(f) = \arctan \frac{(\omega_0 \delta_1 - \omega_1) \sin 2\pi f}{\omega_0 + \omega_1 \delta_1 - (\omega_0 \delta_1 + \omega_1) \cos 2\pi f}; \quad 0 \leq f \leq \frac{1}{2}
\]

which is Equation (12) in the text.

6 NOTATION

\( a \) residual term for average migration rate
\( a' \) residual noise term for bend erodibility
\( A \) scour factor, \( S_L r_o / H \)
\( b \) channel half width
\( B \) backward shift operator
\( c^* \) dimensionless centerline curvature deviation, \( C^* - \overline{C}^* \)
\( C^* \) dimensionless centerline curvature, \( b/r_o \)
\( \overline{C}^* \) average centerline bend curvature
\( C_f \) friction factor, \( gHII/U^2 \)
$E, E_0$  dimensionless coefficient of bend erosion
$ar{E}$ average coefficient of bend erosion
$f$ spatial frequency
$F(f)$ frequency response function
$g$ acceleration due to gravity
$g_s$ steady state gain
$G(f)$ gain function
$H$ centerline depth
$H_0$ value of $H$ in a straight channel
$i$ imaginary number, $\sqrt{-1}$; integer lag
$l$ centerline water-surface slope
$j$ integer lag
$k$ discrete model coefficient, $E_0 \chi$
$k'$ discrete model coefficient, $E_0 \chi(A + 2)(1 - \phi')/2$
$k_t$ discrete weighting coefficient
$K$ constant curvature value
$\hat{I}$ local centerline coordinate
$l$ dimensionless local centerline coordinate, $\hat{I}/H_0$
$N_r$ residual noise term for local migration rate
$P(f)$ phase function
$r_0$ centerline radius of curvature
$s$ centerline coordinate
$s$ dimensionless centerline coordinate, $\hat{s}/H_0$
$S$ centerline arc length
$S$ dimensionless arc length, $\hat{S}/H_0$
$S_L$ cross-channel bed slope
$u$ moving-average order
$U$ centerline vertically-averaged streamwise velocity
$U_0$ value of $U$ in a straight channel
$v$ autoregressive order
$\alpha$ autoregressive coefficient in erodibility model (4.2)
$\beta$ bend angle, defined in Figure 1.1
$\gamma$ autoregressive coefficient in curvature model (9)
$\delta$ autoregressive coefficient in difference equation (7)
$\Delta$ spatial increment operator
$\varepsilon$ moving-average coefficient in curvature model (9)
$\hat{s}$  local channel migration rate
$\xi$  dimensionless local channel migration rate, $\hat{s}/U_0$
$\bar{\xi}$  average bend migration rate
$\phi^j$  discrete autoregressive coefficient, $\exp(-2\chi C_j \Delta s j)$
$\chi$  dimensionless streamwise velocity, $U/U_0$
$\theta$  central angle associated with $\hat{S}$
$\theta'$  dimensionless central angle associated with $S$, $b\theta/H_0$
$\psi$  arbitrary angle
$\eta$  moving-average coefficient in erodibility model (4.2)
$\Lambda$  autoregressive quantity, defined by Equation (13)
$\omega$  moving-average coefficient in difference equation (7)

7 REFERENCES CITED


FIGURE CAPTIONS

Figure 1.1. Illustration of steady-state gain g, as output in migration rate eventually reached in response to a unit step input in curvature $C^*$; this physically coincides with constant-curvature bend preceded by a long straight reach. Compare with Figure 2 in text.

Figure 1.2. Variations of curvature $C^*$ and migration rate $\xi$ with distance $s$ along four isochrones associated with bends one and two, as mapped by Hickin and Nanson (1975).

Figure 2.1. Illustration of piecewise continuous response of migration rate $\xi$ to a pulsed curvature input $C^*$.

Figure 2.2. Smooth curvature series $C^*$ approximated as series of pulses.
FURBISH 2.1

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