Fractal Aspects of Geomorphic and Stratigraphic Processes
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ABSTRACT
Fractal behavior implies power-law, scale-invariant statistics; these statistics are applicable to a wide variety of geological problems. Although topography is often complex, statistically, it usually exhibits fractal behavior; drainage networks are a classic example of a fractal tree. High stands and low stands in reservoirs have been demonstrated to obey fractal, power law statistics; there is also evidence that peak river discharges during floods obey fractal statistics. Floods are responsible for the deposition of some sedimentary sequences; sedimentary layering under a variety of circumstances satisfies fractal statistics.

INTRODUCTION
The subjects of hydrology, geomorphology, sedimentology, and stratigraphy are interrelated. Landforms are created by tectonic processes but are destroyed by erosion; sediment erosion and deposition are primarily responsible for the surface morphology of the continents. Erosion is dominated by floods, but it is a subject of controversy whether erosion is dominated by the very largest floods or by a characteristic flood, say the 10 year or 100 year flood. Erosional processes are responsible for the development of drainage networks, which in turn dominate the development and evolution of landforms. The composition of sediments is responsible for the development of stratigraphic sequences. These sequences may contain a wealth of information on paleoweatther and climate if they can be interpreted.

It is easy to argue that the coupled processes of rainfall, runoff, erosion, material transport, and deposition are so complex as to defy analysis. Yet it must be recognized that there is considerable order in this complexity; fractal statistics are widely applicable. Self-similar fractals are defined by the relation

\[ N = C D^{D} \text{ or } D = \log (N/N_{A}) \log (L/L_{A}) \]  

where \( D \) is the fractal dimension, \( N \) is the number of objects with a linear dimension \( r \), and \( C \) is a constant of proportionality. The concept of fractals was introduced by Mandelbrot (1967) in terms of the length of the west coast of Great Britain. His result is given in Figure 1A; the measured length \( L \) of the coast line is given as a function of the length \( r \) of the measuring rod. Good agreement with the fractal relation

\[ L = N \]  

is found taking \( D = 1.25 \). Similar results are obtained for the length of contours on topographic maps; three examples are also given in Figure 1 for diverse geologic settings. It is seen that there is little variation in the fractal dimension (1.5-1.25); the fractal dimension of topography is not sensitive to the geologic setting and is not diagnostic of age.

The height of topography along a linear track is equivalent to a time series. It is common practice to expand a time series in a Fourier series over the interval \( t \); the coefficients in the Fourier series \( A_{n} \) correspond to the wavelength \( \lambda_{n} = 2\pi r_{n} \) with \( r_{n} \) the wave number. The spectral power density of a time series is given by \( S_{n} = A_{n} \lambda_{n}^{2} \). A time series is a self-affine fractal if (Turcotte, 1992)

\[ S_{n} = C_{3} k_{3}^{D} \text{ or } A_{n} = C_{3} \lambda_{n}^{D-2} \]  

and \( D = 1.25 \). Spherical harmonic expansions of the topography on a planet are equivalent to the Fourier expansion of a time series. Spherical harmonic expansions of the global topography of Earth (Bopp, 1989) and Venus are given in Figure 2. In both cases, good agreement with equation 3 is obtained taking \( D = 2.0(0.5-1.5) \); this is equivalent to the result \( A_{n} = C_{3} \lambda_{n}^{D} \). In the spectral domain, mountains have the same height to width ratios independent of size.

From Figure 2 it is seen that the amplitude of the topography on Venus, \( A_{n} \), is about a factor of four less than on Earth; but on both planets the topography has the same spectral dependence \( D = 2 \). This is somewhat surprising because erosion is dominant in the evolution of many landforms on Earth; erosion is virtually absent on Venus, so that tectonic processes are dominant. This suggests that the tectonic processes that build topography and the erosional processes that destroy topography both give the same statistical behavior. Once again, the fractal dimension of topography does not appear to be diagnostic. It should also be pointed out that the fractal dimensions of topography associated with self-affine spectral expansions and self-similar coastline lengths are not, in general, equal. However, under a wide variety of conditions, topography does obey fractal statistics to a good approximation.

Drainage networks are classic examples of fractal trees. It is standard practice in geomorphology to use the Strahler (1957) ordering system. When two first-order stream segments combine to form a second-order stream, they form a stream with one higher order than the original. Thus, two first-order streams combine to form a second-order stream, two second-order streams combine to form a third-order stream, and so forth. The bifurcation ratio \( R \) is defined by

\[ R = \frac{N_{n}}{N_{n+1}} \]  

where \( N_{n} \) is the number of streams of order \( n \). The length-order ratio \( L \) is defined by

\[ L = \frac{f_{n+1}}{f_{n}} \]  

and \( f_{n} \) is the mean length of streams of order \( n \). From equation 1 the fractal dimension of a drainage network is (LaBarbera and Rosso, 1989)

\[ D = \log R \]  

Horton's (1945) laws require that \( R_{n} \) and \( R_{h} \) be nearly constant for a range of orders \( n \) in a drainage basin; thus, drainage networks were recognized as

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FLOOD STATISTICS

An important question in geomorphology concerns which floods dominate erosion. Is erosion dominated by the 10-year, the 100-year, or the very largest floods? The answer to this question depends upon whether extreme flood probabilities have a logarithmic or power-law dependence on time. The peak river discharge $V$ during a flood is a measure of its intensity. If floods have a logarithmic dependence on time, the peak discharge $V$ during the most severe flood in a time interval $T$ depends on $T$ according to

$$V = C_1 \log T + C_2.$$  

(7)

If floods have a power-law (fractal) dependence on the interval, then we have

$$V = C_3 T^{\gamma}.$$  

(8)

With the logarithmic dependence, extreme floods are much less likely to occur than with the power-law dependence. Thus, the more frequent, intermediate-size floods will carry the bulk of the eroded sediment whereas the rare extremely large flood. With a power-law dependence, the very largest floods are generally responsible for the bulk of sediment transport.

Flood frequency statistics also have a variety of other implications; land-use regulations and flood control projects are based on extrapolations for future floods.

Records of the peak flood discharges are generally available for a relatively short period of time; typically 50 to 100 years in the United States. The objective of flood-frequency analysis is to extrapolate the historical record to longer periods of time. A wide variety of statistical distributions have been utilized for this purpose; Tucomte and Greene (1993) have suggested the applicability of the fractal relation in equation 8. The fractal distribution can also be expressed in terms of the ratio $F$ of the peak discharge over a 10-year period to the peak discharge over a one-year interval. With self-similarity, the parameter $F$ is then also the ratio of the 100-year peak discharge to the 10-year peak discharge and the 1000-year peak discharge to the 100-year peak discharge. The parameters $H$ and $F$ can be related by

$$F = 10^H.$$  

(9)

We refer to the parameter $F$ as the flood intensity factor.

As two specific examples we consider station 1-1805 on the Middle Branch of the Westfield River in Goss Heights, Massachusetts, for the period 1911-1960 and station 11-0980 in the Arroyo Seco near Pasadena, California, for the period 1914-1965. These stations were chosen because they were two of the ten stations used as benchmarks by Benson (1968), who applied a variety of geostatistical distributions to flood-frequency forecasting. The benchmarks are considered independent only if the peak flows are separated by more than one month. For a 50-year record, 50 largest values of $V_i$ are ordered, the largest $V_{50}$ is assigned a period $T = 50$ yr, the second largest flood a period $T = 50/2 = 25$ yr, the third largest flood a period $T = 50/3 = 16.7$ yr, and so forth. The results for the two stations are given in Figure 2.

For station 1-1805 the fractal fit (F) gives $H = 0.51$ and $F = 3.3$. For station 11-0980 the fractal fit gives $H = 0.87$ and $F = 7.4$. Also included in Figure 3 are the six statistical correlations given by Benson (1964):

- For the Pearson type I (P)
- For the Pearson type II (II)
- For the Pearson type III (III)

The Pearson type II (II) is the federal government distribution for evaluating the flood hazard in the United States. For station 1-1805 the 100-year flood predicted by the fractal correlation is a factor of 1.6 greater than the 100-year flood predicted by the log Pearson type III correlation. For station 11-0980 the 100-year flood predicted by the fractal correlation is a factor of 2.3 greater than the 100-year flood predicted by the log Pearson type III correlation. The Pearson type III statistics consistently underestimate the severity of the 100, 150, and 200 year floods.

The values of $H$ and the flood intensity factor $F$ for the ten benchmark stations considered by Benson (1968) are given in Table 1. These results show that there are clear regional trends in the values of $F$. The values in the southwest are systematically high; this can be attributed to the arid conditions and the rare tropical storm that causes severe flooding. The values in the Pacific northwest are low; this can be attributed to the maritime climate. Because $F$ is equivalent to a fractal dimension, $D = 2 - log F$, this may be a case in which the fractal dimension of floods is diagnostic of climate.

The flow in a river is equivalent to a time series. The sum or integral of the flow in a river gives the volume of water stored in a reservoir. Harald Hurnt spent his life studying reservoir storage on the Nile and concluded that extreme high stands and low stands in reservoirs obey power-law (fractal) statistics (Hurnt et al., 1965). The relations between Hurnt's work and self-affine fractals have been considered in detail by Mandelbrot and Wallis (1969a, 1969b).

DRAINAGE NETWORKS

Floods cause erosion, and this erosion eventually forms drainage networks. An example of a drainage network is given in Figure 4A; this is the drainage network in the Vole and Beld Canyons, San Gabriel Mountains, near Glendora, California, obtained by field mapping (Maxwell, 1960). On average, one lower order of streams was found than on the underlying topographic maps; thus, the lowest order streams are assigned order 0. The number-length statistics for this network are given in Figure 5A; a good correlation with equation 6 was obtained taking $D = 1.81$. Leopold et al. (1964) have obtained similar results for the entire United States; a good correlation with equation 6 was also found with $D = 1.83$. Drainage networks are in general fractal, with little variation in the fractal dimension. Again, the fractal dimension of the drainage network is not diagnostic of its geologic setting. Various statistical models were proposed in the 1960s in order to simulate drainage networks; this work was reviewed by Smart (1972). In the past...
three years there has been a rebirth of interest in the problem. Various models have been proposed by Willgoose et al. (1991), Mazikin et al. (1991), Stark (1991), Chase (1992), Kramer and Marler (1992), and Inaoka and Takaya (1993). As a typical example, we consider the diffusion-limited aggregation (DLA) model proposed by Mason and Tuttle (1993). We consider a square grid of points and introduced seed cells on the boundary of the grid. The mechanics of the model are illustrated in Figure 6. A square grid of 15 x 15 cells is used in this illustration. Five seed cells are introduced at random points on the lower boundary. The evolving network must grow from these seed cells. For the example shown, 16 cells have been accreted to the seed cells. Cells are allowed to accrete if one (and only one) of the four nearest neighbor cells is part of the preexisting network. Prohibited sites that already have two occupied neighboring sites are identified by stars. Sites available for accretion to the network are indicated by open circles. A random walker is initiated at a random cell on the grid, and the hypothetical path is traced by the solid line. After 28 random walks it accretes to the network at the shaded cell. A random walker proceeds until the walker exits the grid, or (3) lands on a prohibited cell. In each case the walk is terminated, and a new walker is introduced.

Figure 4. A: The drainage network in the Volte and Bell Canyons, San Gabriel Mountains, near Glendora, California, obtained from field mapping. B: Illustration of a DLA model for a drainage network.

Figure 5. Dependence of the number of streams of various orders on their mean length for (A) the example illustrated in Figure 4A and (B) the model illustrated in Figure 4B. Each circle corresponds to the stream order indicated and the correlations are with equation 8.

Figure 6. Illustration of the mechanism for network growth in the diffusion-limited aggregation (DLA) model. A random walker is randomly introduced to an unoccupied cell. The random walk proceeds until a cell is encountered with one (and only one) of the four nearest neighbors occupied (striped cell). The new cell is accreted to the drainage network, other allowed and prohibited sites are shown.

Figure 7. STATISTICS OF SEDIMENTARY LAYERS

Eroded sediments are eventually deposited as part of a layered sedimentary sequence. Each layer represents a distinct sedimentation event with an upward gradation from coarse-grained sediments to fine-grained sediments; individual layers are generally separated by well-defined bedding planes. Sediment deposition is a very complex process of series of events. In some settings sediments are deposited directly by floods, as in deep lakes. In these cases it may be possible to infer flood-frequency statistics and paleoestimation from sedimentary layering statistics. Sediments deposited in shallow water can be transported and redeposited by storms. Despite the complexities, sediment layering under a variety of circumstances exhibits fractal statistics. Two recent studies of the thickness statistics of turbidite deposits show fractal statistics. Rothman et al. (1994) carried out direct measurements on an outcrop of the Kingston Peak Formation near the western end of Death Valley, California. Their results are given in Figure 8A; an excellent correlation with equation 1 is obtained taking $D = 1.39$. Hiscott et al. (1992) have studied a volcanioclastic turbidity-current deposit in the Bonin forearc basin offshore of Japan. Layer thicknesses were obtained from formation-microresonant images in the upper Oligocene part of the section. Results for two DSDP holes located 75 km apart are given in Figure 8B; a good correlation with equation 1 is obtained taking $D = 1.12$.

Turbidite deposits are associated with slumps off the continental margin. Although it is difficult to determine the thicknesses of turbidite deposits, it is likely that the fractal distribution of layer thicknesses implies a fractal size distribution of slumps. It is interesting to note that Rothman et al. (1994) introduced the concept of self-organized criticality in terms of the size distribution of sand slides off sand dunes. A conclusion of their study was that the size distribution of sand slides should be fractal. Over the past several years, several laboratory studies have been carried out to determine the circumstances under which sand slides exhibit fractal statistics; this work was reviewed by Nagel (1992). The fractal statistics of turbidite layering is evidence that the associated slump formation may be an example of self-organized criticality.

Fractal correlation of sedimentary sequences is not restricted to turbidite deposits. Stollum (1991) obtained fractal statistics for the Middle Jurassic Tilfo Formation in the North Sea Halten Bank basin; these sediments were deposited in a marine shelf environment. Stollum found values of $D = 0.71$, 0.80. Malamud and Tuttle (1992) obtained fractal statistics for sand-bed thicknesses in the shallow marine environment of the Late Devonian Ithaca Formation, New York, with $D = 1.41$. Thus, the fractal behavior of stratigraphic sequences has also been demonstrated using spectral techniques. Hewett (1986) gave results for a density-porosity log in a well through a late Miocene-early Pliocene sandstone formation deposited in a deep submarine fan. He showed that the spectrum of the well log correlated with equation 3, taking $B = 0.71$. Similar results have been reported by Todeschini and Kowalski (1988) and by Todeschini et al. (1990).

Hewett (1986) also developed a fractal-based interpretation technique for determining the porosity distribution in reservoirs. The three-dimen-
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Fundamental geomorphic and stratigraphic processes are relatively poorly understood. Some might say that the underlying processes of flood- ing, erosion, sediment transport, and stratigraphy are so complex as to defy successful modeling. Yet it is recognized that these processes satisfy fractal statistics under a wide variety of circumstances. There is a strong suggestion that modern approaches in statistical physics may be applicable to this class of problems. Examples include diffusion-limited aggregation and self-organized criticality; these approaches yield fractal statistics. In these problems, past processes were commonly addressed by geostatistical empiricism. Fractal frequency analysis is an example. It is well exciting to see whether some or all of these processes can be modeled by the new approaches.

In addition, there is the suggestion that stratigraphic layering may contain a wealth of unused information. If the fractal dimension of floods is climate dependent and if stratigraphic sequences can be correlated directly with floods, then the sequences may provide improved data for the evaluation of the flood hazard today as well as providing a new database for palaeoclimatology.

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REFERENCES CITED


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