Satellites of giant planets are typically tidally locked, with the same side constantly facing the planet. Stronger gravitational forces on the planet facing side cause a permanent tidal bulge to form in the satellite, where the height of the tidal bulge is given by:

\[ H = h_2 R_s \left( \frac{M_p}{M_s} \right) \left( \frac{R_s}{a_s} \right)^3, \]  

(1)

where \( R \) is radius, \( M \) is mass, \( a \) is semi-major axis, the subscripts \( s \) and \( p \) represent the satellite and the planet, and \( h_2 \) is the love number which describes the radial response of the satellite to the changes in gravitational potential. For a homogeneous satellite \( h_2 = (5/2)/(1 + (19\mu)/(2\rho g R_s)) \), where \( \mu \) is the shear modulus, \( \rho \) is the density, and \( g \) is the gravity of the satellite (Love 1944).

If the satellite’s orbit about the planet is eccentric, the size of the tidal bulge will increase when the satellite is closer to the planet, and the satellite’s orbital velocity will also increase, causing the tidal bulge to become misaligned with the planet, resulting in a gravitational torque on the satellite (See Burns 1986). These effects cause the satellite to be repeatedly flexed and strained, where the height of the tidal bulge varies by approximately \( 3eH \). For Miranda the time varying tidal bulge is approximately 0.3 meters for an eccentricity of \( e=0.05 \), and the approximate tidal strain \( \varepsilon \approx 3eH/R_s = 1 \times 10^{-6} \).

The mechanical response of the satellite’s interior to these strains is dependent on the rheological properties of ice, which will control what portion of the tidal energy is partitioned elastically, plastically and anelastically (see e.g. McCarthy Castillo-Rogez 2013). Strain energy partitioned viscously is largely converted to heat. The Maxwell model prescribes a viscous and elastic response in series, and is one of the simplest and most commonly used dissipation models for icy satellites (see Ojakangas and Stevenson 1989). The Maxwell model predicts the amount of heating per unit volume (Showman and Han 2004), in a satellite is

\[ q = \frac{\varepsilon^2 \omega^2 \eta}{2 \left[ 1 + \eta \omega^2 / \mu^2 \right]}, \]

(2)

where \( \omega \) is the satellites orbital frequency. Energy dissipation maximizes when the Maxwell time \( \eta / \mu \) is equal to the orbital forcing period. For Miranda, this model predicts a maximum of 2 – 3 GW can be generated when \( \eta = 10^{14} \) Pa s, close to the viscosity of ice near the melting point. This suggests that tidal resonance could have generated significant power, but only if Miranda was already warm. However, the Maxwell model does treat anelasticity and so may under predict the amount to dissipation for larger viscosities (McCarthy and Castillo-Rogez 2013).
Because the physical mechanism controlling dissipation in ice is poorly understood, a simpler approach may be more practical. The magnitude of tidal heating can be written as

\[ \dot{E} = e^2 \frac{k_2}{Q} \frac{n^5 R^5}{G} \frac{21}{2} \],

where \( R \) is the satellite radius, \( n = 5.15 \times 10^{-5} \) is the orbital frequency, \( G \) is the gravitational constant, \( e \) is the eccentricity, \( k_2 \) is the tidal love number and \( Q \) is the dissipation factor of the satellite (Peale & Cassen 1978). The tidal love number, which describes the satellite's rigidity, is calculated as

\[ k_2 = \frac{3/2}{1 + \frac{19\mu}{2\rho g R}} \],

where \( \mu \) is the rigidity [Love 1944]. We assume \( Q = 20 \) and \( \mu = 4 \times 10^9 \) Pa, the same values used for Enceladus by Meyer and Wisdom [2007], and calculate \( k_2 = 9.265 \times 10^{-4} \). This yields a tidal dissipation rate of 5 GW for an eccentricity of 0.05.

An important consideration is that the time-averaged rate of energy dissipation in a satellite is limited by the amount of energy that can be extracted out of the parent planet (Meyer and Wisdom 2007). That rate depends on the torques exerted by each satellite, \( T \), which depend on the \( k_2 \) and \( Q \) of the planet. This rate is given by Meyer and Wisdom [2007],

\[ \dot{E} = \frac{n_m T_m}{\sqrt{1-e_m^2}} + \frac{n_u T_u}{\sqrt{1-e_u^2}} \frac{T_m+T_u}{L_m+L_u} \left( \frac{G M m_m}{a_m} + \frac{G M m_m}{a_m} \right), \]

where \( L \) is the angular momentum, \( a \) is the semi major axis, \( M \) is the mass of Uranus, \( m \) is the mass of the satellite, and the subscripts \( m \) and \( u \) represent Miranda and Umbriel. We use \( Q_p = 11000 \) (Tittemore and Wisdom 1990) and \( k_{2p} = 0.104 \) (Gavrilov and Zharkov 1977), and calculate that the equilibrium rate of tidal heating is 0.3 GW.
Thus for tides to generate significant power of ~5 GW, Miranda’s eccentricity must be excited well above equilibrium (e.g. Dermott et al., 1988). The eccentricity would then be expected to dampen. Higher dissipation rates lead to more rapid damping of eccentricity. Dermott et al., (1988) calculated the time scale of eccentricity damping on Miranda to be $t_{\text{damp}} = 2 \times 10^6$ Q years, where Q is the quality factor of Miranda. So for Q = 20, the damping timescale is 40 million years. In almost all of our convection simulations, convection begins deforming the surface in less that 0.1 diffusion time scales. The diffusion time scale is $t_{\text{diff}} = \frac{D^2}{\kappa} \sim 100$ Myr. Therefore the timescale of convective surface deformation and eccentricity damping are approximately the same, suggesting the convection driven resurfacing could form the coronae in the time it takes for eccentricity to dampen.

**Item DR2: Critical Rayleigh Number**

The critical Rayleigh number for convection in a Newtonian fluid is $Ra_{cr} = 20.9 \theta^4$, where \( \theta = Q \Delta T / RT_i^2 \) [Solomatov 1995]. For pure ice deforming by volume diffusion, Q=59.6 kJ/mol. Using the same $\Delta T$ and $T_i$ as in the methods section, $Ra_{cr} = 3 \times 10^7$ for pure ice in Miranda’s ice shell. However, we imagine that tidal flexing during orbital resonance may have weakened the lithosphere prior to the initiation of convection and that the effective viscosity contrast across the surface was limited by the yield stress of the lithosphere. When convective stresses exceed the yield strength of the lithosphere $\sigma_y$, the effective viscosity of the surface $\eta_0$, can be limited to $\eta_0 = \sigma_y / 2 \dot{\varepsilon}$ (e.g. Tackley 1993, Showman and Han 2005), where $\dot{\varepsilon}$ is the surface strain rate. For a yield stress of 10 kPa, similar to the apparent yield stresses of the surfaces of Europa and Enceladus (see Hoppa et al., 1999, Hurford et al., 2007), and for typical convective strain rates between $10^{-15} - 10^{-13}$ s$^{-1}$, this yields surface viscosities $\eta_0 = 10^{17} - 10^{19}$ Pa s. This yields a viscosity contrast across the ice shell of $\Delta \eta \approx 10^3$, $\theta \approx 6.908$, and a critical Rayleigh number $Ra_{cr} \approx 5 \times 10^4$. Additionally, Solomatov (2004) calculated the critical yield stress for the initiation of convection-driven resurfacing as

$$\sigma_{y,cr} = \frac{13 \alpha g \rho (RT_i^2 \kappa / \varepsilon)^2}{\Delta_T} l_{\text{hor}},$$

where $l_{\text{hor}} = 300$ km is the width of convective upwellings and $E=59.6$ kJ mol$^{-1}$ K$^{-1}$ is the activation energy of diffusion creep. For an internal temperature $T_i = 200$ K, $\sigma_{y,cr} = 5$ kPa. Tidal flexing during resonance could have fatigued the lithosphere (see Suresh 1999) and lowered the yield stress of the surface to near this value. Additionally, partial melts of ammonia-water ice could generate additional composition buoyancy stresses, limiting the thermal buoyancy stresses required for convection to deform the surface.

**Item DR3: Convection Simulations**

We use the 3D spherical convection model CitcomS (Zhong et al., 2000, Han et al., 2006) to simulate convection in Miranda’s ice shell for a wide range of internal structures, ice viscosities, and heating rates to show that convection can create the thermal gradients, deformation pattern, and global distribution of coronae.

We assume Miranda is at least partially differentiated with a core size of $x = R_{\text{core}} / R_{\text{satellite}} = 0.25, 0.4, 0.55$. These core sizes are computed using...
where $\rho_b = 1.2 \text{ g/cm}^3$ is the bulk density of Miranda (Jacobsen et al., 1992), $\rho_r = 2.5 - 3.5 \text{ g/cm}^3$ is the density of silicate rock and $\rho_m$ is the density of the ice mantle. For the differentiated cases $\rho_m = \rho_i$, where $\rho_i = 0.92 \text{ g/cm}^3$ is the density of ice. For the partially differentiated case $\rho_m = 1.178 \text{ g/cm}^3$, corresponding to a 10% volume fraction of silicates.

Table DR1. Parameters for compositional models

<table>
<thead>
<tr>
<th>Interior Model</th>
<th>$\rho_r (\text{kg/m}^3)$</th>
<th>$\rho_i (\text{kg/m}^3)$</th>
<th>$x$</th>
<th>$\rho_m (\text{kg/m}^3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-density rock</td>
<td>2500</td>
<td>920</td>
<td>0.55</td>
<td>920</td>
</tr>
<tr>
<td>High-density rock</td>
<td>3500</td>
<td>920</td>
<td>0.4</td>
<td>920</td>
</tr>
<tr>
<td>Partially differentiated</td>
<td>3500</td>
<td>920</td>
<td>0.25</td>
<td>1178</td>
</tr>
</tbody>
</table>

The Rayleigh number,

$$Ra = \frac{\rho_i g \alpha (T_b - T_s) D^3}{\kappa \eta_1},$$

(8)

governs the vigor of convection in the ice shell, where $\rho_i = 920 \text{ kg/m}^3$ is the density of ice, $g = 0.083 \text{ m/s}^2$ is gravity, $D = (1 - x) \times R_{\text{satellite}}$ is the ice shell thickness, $T_b$ is the temperature at the base of the ice shell, the surface temperature $T_s = 60 \text{ K}$, $T_i$ is the average internal temperature of the ice shell, the thermal conductivity $\alpha = 1.56 \times 10^{-4} \text{ (T_i/250 K) K}^{-1}$ (Kirk and Stevenson 1987), thermal diffusivity $\kappa = 1.47 \times 10^{-6} \text{ (250 K/T_i)}^2 \text{ m/s}^2$ (Kirk and Stevenson 1987), and the viscosity of ice at the base $\eta_1$ is a function of $T_b$ (Goldsby and Kohlstedt 2001). We find that the average internal temperature increases with increasing core fraction as $T_i \approx T_s + \frac{x}{1-x} \Delta T$, similar to scaling relationships derived from studies of isoviscous convection in spherical geometry (Deschamps et al., 2010), however, we also find that the internal temperature increases with increasing viscosity contrasts (Schubert 2001). On average, $T_i = T_s + 0.3 \Delta T$. For a temperature at the base of the ice shell $T_b = 220 - 270 \text{ K}$, the viscosity of ice deforming by volume diffusion, for a grain size $d = 0.2 \text{ mm}$, $\eta_b = 10^{14} - 10^{17} \text{ Pa s}$ (Goldsby and Kohlstedt 2001), yielding $Ra = 10^{3.5}$ to $10^{7.5}$. 

$$x = \left( \frac{\rho_b - \rho_m}{\rho_r - \rho_m} \right)^{1/3}$$

(7)
These Rayleigh numbers require relatively high temperatures at the base of the ice shell and relatively small grain sizes. Such a grain size may be a plausible if grain growth is restricted by pinning from silicate particles (Barr and McKinnon 2007). The grain size and temperature constraints may be relaxed if ammonia is present in the ice shell. Ammonia hydrate may reduce the viscosity of ice above 173 K. Durham et al. (1993) examined the rheology of the ammonia-water ice system. Melted ammonia-hydrate along ice grains could lower the activation energy of grain boundary diffusion, thereby reducing viscosity. This mechanism may be similar to enhanced deformation rates observed for pure ice above 253 K. Goldsby and Kollhstedt (2001) argue that pre-melting along grains can enhance creep by a factor of 1000. Thus ice with even a small amount of ammonia may have a much lower viscosity above 173 K.

Throughout the ice shell, viscosity varies with temperature as $\eta(T) = \eta_1 \exp(-\gamma T)$, where $\gamma = \theta / \Delta T$, and $\theta = \ln(\Delta \eta)$, where $\Delta \eta$ is the ratio between the viscosity at the surface of the ice shell and its base. The viscosity of ice at the surface is limited to $\Delta \eta = 10^3$ to $10^4$ times the viscosity of ice at the melting point, to mimic the effect of surface ice with a low yield stress (e.g., Trompert & Hansen 1996, Moresi & Solomatov 1998).

The thermal gradient at the surface is calculated as $\frac{dT}{dz} = \frac{\Delta T}{D} Nu$, where $Nu$ is the Nusselt number. Surface deformation rates are calculated as $\dot{\epsilon} = det \left[ \frac{\partial u}{\partial x} \right]$, where $u$ is surface velocity, $x$ is direction and $i,j=1,2$ are indices representing directions of colatitude and col longitude. We run our simulations until the average Nusselt number stabilizes to within 3 significant digits. We compared our results to previous studies by Zhong et al., (2008) and Ratcliff et al., (1996), as well as to scaling relationships for sluggish-lid convection (Barr 2008) derived from convection simulations using the 2D model CITCOM (Moresi and Solomatov 1995). Our results match the values of previous studies and expected values from scaling laws, to within 5-10%.
Figure DR3: The y-axis shows the average Nusselt numbers of our convection simulations after the Nusselt number approaches a stable value. The x-axis shows the expected Nusselt number based on the scaling relationship from Barr (2008), \( Nu = 0.33 (Ra_b)^{1/3} \exp(\theta/19) \), where \( \theta = \ln(\Delta \eta) \), \( Ra_b \) is the Rayleigh number at the base of the ice shell, and \( \Delta \eta \) is the viscosity contrast between the top and the bottom of the ice shell. Circles represent the results of this study, where circle size scales with the size of the silicate core, and plus signs and crosses represent the results of Ratcliff et al., (1996) and Zhong et al. (2008), respectively.

References:


Zhong, S., McNamara, A., Tan, E., Moresi, L., & Gurnis, M., 2008, A benchmark study on mantle convection in a 3-D spherical shell using CitcomS. *Geochemistry, Geophysics, Geosystems*, 9(10).