Supplementary Material

Supplementary Figure DR1. Spatial evolution of the Ruapehu 2007 lahar wave, with discharge shown as a function of arrival time at six stations between 7.7 and 145 km from the Crater Lake dam source. The lahar wave remained a kinematic form until 107 km before dissipating over the remaining c. 90 km travel to the Tasman Sea.

The orange bar depicts the joined peak of linear momentum and energy (both per unit length of lahar). Note the progressive divergence of initially coinciding peaks in discharge and momentum with travel distance to at least 107 km from the Crater Lake dam source.
<table>
<thead>
<tr>
<th>Instrument; Method</th>
<th>Measured Parameter</th>
<th>Sampling Frequency / Resolution</th>
<th>Deployment</th>
<th>Deduced parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acoustic Flow Monitors (single-band geophones with AFM board; e.g. Hadley and Laflusen, 1995)</td>
<td>• Real-time seismic amplitude in two separate (high and low gain) channels</td>
<td>5 seconds / &lt;1-100 Hz</td>
<td>• on river bank</td>
<td>• lahar arrival time $T_A$ (<em>) and lahar front velocity $U_A$ (</em>) through differentiating best-fit power laws of all $T_A$ vs. distance data</td>
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<tr>
<td>Georal CGG-6TD 3-component broadband seismometers</td>
<td>• ground vibrations up to Nyquist frequency (62.5 Hz) • seismic amplitude • frequency response • signal directionality</td>
<td>Down-sampled to 10 seconds / 0.033-60 Hz</td>
<td>• on river bank, N-axis aligned parallel to riverbank</td>
<td>• lahar arrival time $T_A$ (<em>) and lahar front velocity $U_A$ (</em>) through differentiating best-fit power laws of all $T_A$ vs. distance data</td>
</tr>
<tr>
<td>Digital Video Cameras; Particle-Image Velocimetry on paired images</td>
<td>• Mid-channel surface velocity $U_S$ (e.g. Doyle et al. 2011)</td>
<td>Paired images at 25 fps every 30 seconds / 0.05 m/s</td>
<td>• pole- or tripod-mounted at bridges and riverbanks</td>
<td>• body velocity $U_{B}$ (*) assuming a logarithmic velocity profile and the relationship $U_{B}=kU_{0}$, where $k=0.75$ (e.g. Doyle et al. 2011)</td>
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<tr>
<td>Gas bubbler</td>
<td>• Crater Lake Level</td>
<td>6 seconds</td>
<td>• in Crater Lake</td>
<td>• Initial dam-break hydrogram (*) through numerical modeling of flow through a broad-crested weir and constraining breach growth through automated still camera (Manville 2001)</td>
</tr>
<tr>
<td>Ultrasonic and Laser Stage Gauges</td>
<td>• Flow height</td>
<td>30 and 60 seconds, except for 107 and 145 km stations were sampling rate was 300 and 120 seconds, respectively, and intermediate stage values were determined from video</td>
<td>• mounted to bridges at mid-channel positions with datum calibrated to stable (preferably hard rock) pre-lahar cross-sections; larger depositional cross-section changes (&lt;5% of channel capacity) occurred after the interval $(T_P, T_U)$ considered here.</td>
<td>• Wetted perimeter $P$ and wetted area $A$ (<em>) • hydraulic radii $h$ and $R$ (</em>) • lahar arrival time $T_A$ (<em>) and lahar front velocity $U_A$ (</em>) through differentiating best-fit power laws of all $T_A$ vs. distance data • lahar peak flow time $T_0$ (<em>) and lahar peak flow velocity $U_0$ (</em>)</td>
</tr>
<tr>
<td>Pore-pressure Cell</td>
<td>• Flow depth</td>
<td>5 seconds</td>
<td>• mounted into cylindrical holes drilled into solid river bed</td>
<td>• Wetted perimeter $P$ and wetted area $A$ • hydraulic radii $h$ and $R$ • lahar arrival time $T_A$ (<em>) • lahar peak flow time $T_0$ (</em>) and lahar peak flow velocity $U_0$ (*)</td>
</tr>
<tr>
<td>• Time-series dip sampling, incl. • Water chemistry through Ion Chromatography, Atomic Absorption Spectroscopy • Sediment characteristics through Standard Sieving, Laser Particle Sizing, Particle componentry and density analysis</td>
<td>• Sediment weight and volume percentages • Sediment concentration peak time $T_S$ • Sediment grain-size distribution • Water Chemistry (Mg, F, Cl, SO4, Br, pH) • Water conductivity</td>
<td>10 – 600 seconds (depending on flow variation) before lahar arrival, during first 2-4.5 hours after lahar arrival; daily samples for lahar tail during next 5 days</td>
<td>• 2-4 kg time-series dip samples of suspended sediment-water mixtures from upper 1 m of active flow at channel thalweg (e.g. Pierson and Scott, 1985; Scott, 1988; Rodolfi et al., 1996; Dinehart, 1997; Cronin et al., 1997; Lavigne and Suwa, 2004) • samples were filled into plastic bottles, sealed and analyzed in the laboratory</td>
<td>• volumetric Crater-Lake water/stream water ratio (<em>); (Lube et al. 2009; Christenson et al. 2010); Supplementary material “Calculations of bulk energy and bulk linear momentum” • time-variant mass flux of Crater-Lake water $Q_{1x,0}$ (</em>) (Supplementary material “Calculations of bulk energy and bulk linear momentum”)</td>
</tr>
<tr>
<td>Airborne LiDAR</td>
<td>Semi-Continuous digital elevation data</td>
<td>Pre and post lahar / 0.1 m horizontal</td>
<td>• covering first 62 km of Whangaehu River channel</td>
<td>• spatial net-volume geomorphic change (Procter et al. 2010)</td>
</tr>
<tr>
<td>Real-Time Kinetic GPS</td>
<td>• Channel cross-sections • Channel slope S • Post-lahar super elevations • Post-lahar peak flow tide lines</td>
<td>Pre and post lahar / 3 cm horizontal</td>
<td>• at regular intervals for entire Whangaehu River channel</td>
<td>• alternative estimate of peak flow velocity (Chow, 1959; Pierson and Scott 1985) • stage/wetted perimeter, stage/wetted area and stage/hydraulic radii relationships (*)</td>
</tr>
<tr>
<td>Tripwire</td>
<td>Lahar arrival time</td>
<td>1 measurement</td>
<td>• at a single location in upper Whangaehu channel 1.9 km from Crater Lake</td>
<td>• lahar arrival time $T_A$ (<em>) and lahar front velocity $U_A$ (</em>) through differentiating best-fit power laws of all $T_A$ vs. distance data</td>
</tr>
</tbody>
</table>

Supplementary material Table DR1: List of instruments deployed, parameters measured, sample frequency, instrument deployment specifications and deduced parameters to obtain hydrological, sedimentological, kinematic and chemical time-series data for the 18 March 2007 lahar. (*) denotes parameters that were used in this study.
Supplementary Material: Calculations of bulk energy and bulk linear momentum

In Figure 4A we show computed data of bulk flow energy, $E_{TOT}$, of the frontal part of lahar that contains original Crater Lake water of mass $M_0$ from several lahar monitoring stations along the Whangaehu River.

The time (since lahar arrival) when Crater Lake water of mass $M_0$ has passed each location can be determined by integrating the time-variant mass flux of original Crater Lake water $Q_{CLW}$:

$$Q_{CLW} = F_{CLW}Q_M,$$  \hspace{1cm} (1)

where $F_{CLW}$ is the weight percentage of original Crater Lake water in (sediment/stream water/Crater Lake water suspension) lahar dip samples and $Q_M$ is the time-variant lahar mass flux:

$$Q_M = \rho_L Q_V,$$  \hspace{1cm} (2)

$Q_V$ and $\rho_L$ are time-variant lahar discharge and time-variant lahar density (taken as equal to the bulk density of dip samples), respectively. The acidic, highly mineralized and known composition of Crater Lake water immediately prior to the March 2007 lahar (Christensen et al. 2010), together with measurements of water chemistry (through Ion-chromatography and Atomic Absorption Spectroscopy; Supplementary material Table DR 1) and sediment concentration in dip samples, becomes the key for deriving time-series data of $F_{CLW}$ as:

$$F_{CLW} = (1 - SC)\frac{C_L - C_S}{C_{CLW} - C_S}$$  \hspace{1cm} (3)

$SC$ is sediment concentration in weight percent. $C_L$, $C_S$ and $C_{CLW}$ are the concentrations of chloride in dip samples, in stream water before lahar arrival and in the Crater Lake, respectively. Chloride is well suited for quantifying lahar/stream water mixing processes.
because of its high solubility in neutral to acidic aqueous solutions encountered in Ruapehu lahars (e.g. Cronin et al. 1996). Synchronization is required for combining time-series data of $Q_V$, $\rho_L$ and $F_{CLW}$ to yield curves of mass fluxes for stream water, Crater Lake water and suspended sediment against time. All automatically logged data streams (seismic data, automated video and stage heights) were routinely GPS time-stamped. Manually-operated digital video footage and photographs from cameras with internal clocks were corrected to GPS time after the lahar sampling campaign. This also allowed correcting and specifying the lahar dip-sample times as the sampling procedure was captured by video and photographs. The alignment of constant-frequency time-series for $Q_V$ and the non-constant sampling frequency time-series for $\rho_L$ and $F_{CLW}$ from dip-samples was achieved through cubic spline interpolation of timed $\rho_L$ and $F_{CLW}$ data points. Three additional time-series points were computed through cubic spline interpolation for $T_A$ (specifiable through video and seismic data to less than 5 seconds precision), $T_S$ (specifiable through seismic data to less than 5 seconds precision) and $T_P$ (determined through graphical integration of Crater Lake water mass flux against time curves). These time-constraints together with maxima of $Q_V$ and $F_{CLW}$ in the laharic suspension allow approximating the uncertainty in the calculated values of volume of Crater Lake water in the head to be less than 2 percent.

In order to obtain $E_{TOT}$ for each location, we compute the time-variant bulk energy flux, $Q_{E_{tot}}$, as the sum of the time-variant fluxes of potential ($Q_{E_{pot}}$) and kinetic energies ($Q_{E_{pot}}$) through each observation point:

$$Q_{E_{tot}} = Q_{E_{pot}} + Q_{E_{kin}} = gQ_MH + \frac{1}{2}Q_MU_D^2,$$

(4)

$H$ denoting height above sea level. Data of $Q_{E_{tot}}$ against time $t$ can be integrated over the time interval $[t_x, t_{x+1}]$ to yield the bulk energy of the fraction of lahar that passed the location during this time interval:
The total linear momentum \( P(t_x, t_{x+1}) \) of the fraction of lahar that passes a monitoring location between times \( t_x \) and \( t_{x+1} \) (Fig. 3C) can be computed in a similar manner by integrating curves of time-variant linear momentum flux \( Q_P \):

\[
E_{\text{tot}}(t_x, t_{x+1}) = \int_{t_x}^{t_{x+1}} Q_{E_{\text{tot}}}(t) dt. \tag{5}
\]

\[
Q_P = Q_M U_D \tag{6}
\]

and time \( t \) as:

\[
P(t_x, t_{x+1}) = \int_{t_x}^{t_{x+1}} Q_P(t) dt. \tag{7}
\]

**References cited in supplementary material:**


