Appendix A: Evaluating errors in Balanced cross sections

Orogen-scale balanced cross sections are the only tool available for obtaining quantified estimates of shortening. However the purpose of balanced cross sections, to find a solution that is the best fit to the available data, does not easily lend itself to quantifying errors on that solution. A popular solution to this problem, combining all available estimates, magnifies errors by combining estimates with different geographical boundaries and/or different assumptions or equally weighting estimates that are not balanced. For our purpose, the most critical data for determining the magnitude of shortening is the amount of shortening required by exposed surface structures. The central Andes are probably one of the best regions to produce a tightly constrained balanced section because the thick Paleozoic stratigraphy is still preserved over the entire orogen and most of the faults preserve hanging wall cut-offs (in the line of section or along strike) greatly limiting large variations in shortening magnitude. To help evaluate the potential error in shortening estimates though Bolivia, we identified 6 locations in the southern section and 4 in the northern section where eroded hanging wall cut-offs indicate regions in which fault displacement is not explicitly known. Although theoretically shortening along faults where the hanging wall cut off is missing is an unknown, changing the magnitude of displacement on an individual thrust must be accompanied by changes in the subsurface geometry in a way that the section still balances. In Figure 3 we have identified structures where additional or less slip is permissible at the surface, but have not tested the subsurface implications of different displacements. Along the southern cross section there are 6 thrusts that have 3-8 km of displacement between the modern day erosion surface and their respective hanging wall cut-offs (Fig. 3). This material could be viewed as potential “extra” shortening. The cumulative amount of “extra” displacement is 32 km, roughly 10% of the 326 km of total shortening. Subtracting 32 km from the total shortening amount, shortening in the southern section could be as low as 294 km or 35% (Table 1). In the north there are 4 thrusts in which the hanging wall cut-off has been eroded and where additional slip could be accommodated. Assuming the magnitude of extra slip on each fault is similar to what was calculated in the south, we estimate 25 km of additional slip or ~10% of the 276 km of total shortening. Adding 25 km to the total shortening amount suggests shortening in
the northern section could be as high as 301 km or 42% (Table 1). Our error evaluation
highlights two important points: (1) a reasonable error estimate in our balanced section is
~10%, and (2) given this amount of error, it is not possible to tell if propagation has been
limited by erosion or not, suggesting the need for additional shortening and exhumation
estimates. These estimates are discussed in the accompanying manuscript where we
comparing shortening estimates between the balanced section and cross-sectional area
(Fig. 4), as well as compare predicted magnitudes of exhumation to exhumation estimates
from thermochronometers (Table 2).

Appendix B: Comparing calculated and predicted changes in orogen width.

Whipple and Meade (2004) used critical taper theory, the assumption of steady
state topography and stream power fluvial incision models (e.g. Whipple and Tucker,
1999) to relate orogen width to erosional fluxes from the orogen due to precipitation
gradients. For fixed influx and sediment recycling they found:

\[ W_2 = (\tan \alpha_2)^{0.25-1.82} (K_2)^{0.37-0.93} \]
\[ W_1 = (\tan \alpha_1) (K_1) \]

Where \( W \) is orogen width, \( \alpha \) is the taper angle of mean topography, \( K \) is a coefficient of
erosion and represents the factor of 2 difference in the erosivity from north to south (due
to precipitation), and exponents are Typical values of erosion parameters specified in
Whipple and Meade (2004). Using \( \alpha \) (0.75-1.25-2) and \( K \) (1 and 2) appropriate to
southern and northern Bolivia (respectively), analytical models (Whipple and Meade,
2004, 2006), assuming uniform precipitation, predict a 35%-80% reduction in orogen
width.

The original equations outlined by Whipple and Meade (2004) assume a steady-
state configuration of the orogen in response to different climate forcings, rather than the
transient response of the orogen to the changing climate as we suggest in this paper.
Nevertheless, application of the steady-state approach to the previously discussed
geometries and magnitudes of climate change provides a first order comparison between
analytical models and our results. Our calculations suggest that the 2 fold increase in
precipitation in the north limited SA growth (propagation) by ~30%, a value near the lower bounds of analytical models. However, if the assumption of uniform precipitation is released and we assume linearly decreasing precipitation from east to west (see Fig 1c), then analytical models permit a reduction in width as low as 30%.

Spatially variable precipitation and an orogen scale erosion law

To incorporate for non-uniform precipitation (Fig. 1c) into the orogen scaling relationships developed in Whipple and Meade (2004), we develop a simple modification of the effective erosion law so that it is consistent with the prescribed rainfall distribution. The orogen scale erosion law used by Whipple and Meade (2004) is,

\[ E = KA^m S^n. \] (1)

Here the drainage area, \( A \), is a proxy for discharge, \( Q \), under the assumption of a spatially uniform precipitation distribution. In general, for spatially variable precipitation, the flux, as a function of distance away from the drainage divide, can be written as,

\[ Q(x) = \int_{0}^{x} P(x') \frac{dA}{dx'} dx'. \] (2)

where, \( P(x) \) is the precipitation distribution and \( A(x) \) is the power-law scaling for drainage area as a function of downstream distance which can be written in terms of general Hack’s law as, \( A(x) = cx^h \). If the particular rainfall distribution in the Subandes can be approximated as a linear ramp that decreases away from the drainage divide (Figure 1C), \( P(x) = ax \), we can write the flux as,

\[ Q(x) = P_0 ch \int_{0}^{x} x^{'h-1} dx' = P_0 ch \int_{0}^{x} x^{'h} dx' = \frac{P_0 ch}{h+1} x^{h+1} \] (3)

where \( P_0 \) is a reference precipitation rate. Note that for this particular precipitation distribution the accumulated channel flux is proportional to the product of the drainage area multiplied by the along channel distance from the drainage divide. Thus, for the case of a linear downstream increase in precipitation the flux is proportional to \( x^{h+1} \), rather than \( x^h \) for the linear precipitation case. The Whipple and Meade (2004) scaling relationships can be adapted to this particular rainfall distribution using an effective Hack...
exponent, $h = h + 1$ (e.g., equation (3)), to represent the increased downstream flux due to this particular precipitation distribution (Figure B1).

The Hack exponent contributes to the predicted orogen scaling relationships derived by Whipple and Meade (2004). In particular the width of an actively deforming orogen is controlled by overall mass balance and is a function of the mean slope of the orogenic wedge, $\alpha$, the rate at which material is accreted into the wedge, $F_A$, and the effective erosivity of the climate, $K$,

$$W = \left( \tan \alpha \right)^{h+1-q} F_A^{1} K^{1}.$$

So for fixed influx and sediment recycling rates if erosivity and slope were to change, the ratio of the widths of two orogens would be,

$$\frac{W_2}{W_1} = \left( \frac{\tan \alpha_2}{\tan \alpha_1} \right)^{h+1-q} \left( \frac{F_2}{F_1} \right)^{1} \left( \frac{K_2}{K_1} \right)^{1}. \quad (5)$$

Using the values from Whipple and Meade (2004) $h = 1.67 - 2.00$, $m = 0.30 - 1.00$, $m = 0.30 - 1.00$, $q = 0.11 - 0.20$ yields,

$$\frac{W_2}{W_1} = \left( \frac{\tan \alpha_2}{\tan \alpha_1} \right)^{-0.23 - 1.42} \left( \frac{F_2}{F_1} \right)^{0.34 - 0.91} \left( \frac{K_2}{K_1} \right)^{-0.34 - 0.91}. \quad (6)$$

For the case of a linear precipitation ramp the width scaling can be calculated by replacing the geometric Hack exponent with an effective Hack exponent $h = h + 1$. This gives the following width ratio scaling,

$$\frac{W_2}{W_1} = \left( \frac{\tan \alpha_2}{\tan \alpha_1} \right)^{-0.17 - 1.43} \left( \frac{F_2}{F_1} \right)^{0.26 - 0.72} \left( \frac{K_2}{K_1} \right)^{-0.26 - 0.72}. \quad (7)$$

The predictions for both precipitation distributions are shown in Figure B2 under the assumption of identical accretionary influx $F_{A_1} = F_{A_2}$. The region of models that can be explained by the Whipple and Meade theory is shifted up and to the left providing a better fit to the observations presented in the text.
The data presented here have been interpreted in the context of steady-state orogenic wedge models (e.g., Whipple and Meade, 2004). While this is a simple end member scenario, the on-going growth of the sub-Andes may be better described by a model, which incorporates transient behavior. For the climatic conditions associated with Taiwan Whipple and Meade (2006) estimated approach to steady-state topography between 3-4 Myr. However they noted that systems with less erodible rocks and less rainfall (lower K) can have characteristic response times of tens of millions of years. Thus, although it is predicted that orogenic wedges grown and shrink in response to erosional efficiency (climate and rock properties) (Whipple and Meade, 2006), the magnitude of the change in orogen width would vary depending on whether an orogen is growing in response to different climate conditions or whether a steady-state orogen is responding to different climate forcings.

References


Figure B1. Precipitation and flux distributions as a function of normalized distance away from the drainage divide located at $x = 0$. The total amount of precipitation is the same in both cases. However the downstream flux increases faster when the precipitation gradient increases in the same direction as does the along stream accumulated drainage area.
Figure B2. Uniform and linear precipitation width ratios. The shaded areas show the width ratios predicted by the uniform precipitation model and the linear precipitation model presented here. Note that for the linear ramp case $\frac{W_2}{W_1}$ exceeds 0.6 consistent with the observations presented in the text.