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Methodology with variable relative viscosities

The functional that is minimized in the method of Flesch et al. (2001) is:

\[
I = \int_S \frac{1}{\mu} \left[ \bar{\tau}_{\alpha\beta} \bar{\tau}_{\alpha\beta} + \bar{\tau}_{\gamma\gamma}^2 \right] dS + \int_S 2\lambda \left[ \frac{\partial}{\partial x_\beta} (\bar{\tau}_{\alpha\beta} + \delta_{\alpha\beta} \bar{\tau}_{\gamma\gamma}) + \frac{\partial \bar{\sigma}_{zz}}{\partial x_\alpha} \right] dS
\]

(1)

where \(\mu\) is the relative viscosity, \(\bar{\tau}_{\alpha\beta}\) is the vertically integrated horizontal deviatoric stress, \(\bar{\tau}_{\gamma\gamma} = \bar{\tau}_{xx} + \bar{\tau}_{yy}\), \(\lambda\) is the horizontal component of the Lagrange multiplier for the force balance differential equation constraint, \(\bar{\sigma}_{zz}\) is the vertically integrated vertical stress defined in the main paper and \(S\) represents area on the entire Earth’s surface.

Flesch et al. (2001) assumed a constant \(\mu\) equal to 1. In this paper we use a variable value of \(\mu\) to approximate weak plate boundary zones and strong plates. We assume an inverse relationship between strain rates and relative viscosities, \(\mu\). We obtain the relative viscosities of the deforming plate boundary regions, such as the mid-oceanic ridges and subduction zones, by assigning a reference viscosity to the moderately straining mid-Indian ridge, using the relation:

\[
\frac{1}{\mu} = 1 + \left( \frac{1}{\mu_{ref}} - 1 \right) \sqrt{\frac{E^2}{E_{ref}^2}}
\]

(2)

where \(\mu_{ref}\) is the reference viscosity corresponding to a mid-oceanic ridge with a moderate spreading rate, such as the Indian ocean, \(E^2 = 2(\dot{\epsilon}_{\theta\theta}^2 + \dot{\epsilon}_{\phi\phi}^2 + \dot{\epsilon}_{\phi\phi}^2 + \dot{\epsilon}_{\phi\theta} \dot{\epsilon}_{\theta\phi})\), where \(\dot{\epsilon}_{\theta\theta}, \dot{\epsilon}_{\phi\phi}\) and \(\dot{\epsilon}_{\phi\phi}\) are the strain rates from Kreemer et al. (2003), and \(E_{ref}^2\) is the reference value for \(E^2\), corresponding to the value for \(\mu_{ref}\). A plot of relative viscosities (Fig. DR2) shows the lowest viscosities along the mid-oceanic ridges and higher viscosities in
the deforming continents, while the blank areas (the plates) have a $\mu$ value of 1. We try reference viscosities of 0.01, in which the mid-oceanic ridges are 100 times weaker than the plates (Fig. DR3 and the solution in the main paper), and also 0.001, in which the mid-oceanic ridges are 1000 times weaker than the plates. We show the global stress solutions for three cases: the case where $\mu = 1$ everywhere (Fig. DR1) and cases in which $\mu$ varies according to the inverse of strain rate (Figs. DR3 and DR4). The solutions with reference viscosities of 0.01 and 0.001 respectively yield a focusing of stresses within the plates and fits well with the observed $SH_{max}$ orientations (Zoback et al., 1992) in most places (Figs. DR3 and DR4). Moreover, solutions with weak plate boundaries provide agreement of $SH_{max}$ for Indo-Australian plate region (Sandiford et al., 1995). However the N-S deviatoric extension in Tibet remains as a prominent feature in the global solution. We also considered a special case in which the Tibetan Plateau and the Himalayas were assigned viscosities equal to the rigid plate interiors (Fig. DR5). Although there occurs a marginal reduction in the N-S deviatoric stresses within the Tibetan Plateau for this case (Fig. DR6), such a strong resistant plate boundary is insufficient to focus deviatoric stresses to cancel large N-S deviatoric extension associated with excess GPE of Tibet. Moreover, the assumption of plate-like strength of this plate boundary zone is not reasonable, given the high rates of deformation occurring there (see Fig. DR2).

**Dynamic topography**

Actual topography, in some places, already contains a contribution from dynamically induced radial tractions. We argue here that GPE values from the uncompensated
model (Fig. DR7) contain the influence of dynamic topography. In the presence of
dynamic topography, the weight of the lithospheric column, $\sigma_{rr}^{total}$, at reference depth $L$
is $\sigma_{rr}^{total} = \int_{-h}^{L} \rho(r)g(r)dr = \sigma_{rr}^{o} + \tau_{rr}$ (note: this ignores the contribution from flexure),
where $\sigma_{rr}^{o}$ is a reference stress, and $\tau_{rr}$ is the radial traction associated with deeper man-
tle flow that is responsible for dynamic topography. The physical effect of this dynamic
topography is therefore taken into account already in the GPE calculations that are un-
compensated (Fig. DR7), since these involve the vertical integral of $\sigma_{rr}(r)$ down to the
depth $L$. In order to remove the contribution from dynamic topography, an isostatically
compensated solution (uniform pressure, $\sigma_{rr}^{p}$, at depth $L$) was calculated by adjusting
the densities of the sub-crustal part (upper mantle) of each lithospheric column. The
GPE and the resulting stresses were then calculated as usual. Compensation can also
be achieved by adjusting the elevation of each column instead of adjusting the density.
However, since greater uncertainty lies in the values of densities of the upper mantle
than in the values of the crustal thickness, compensation obtained by density adjust-
ment seems to be more reasonable than that obtained by adjusting the elevation. As
stated earlier, the uncompensated solution only contributes 20% higher stresses. The
Tibetan plateau shows vertically integrated deviatoric extension of the order $\sim 3 \times 10^{12}$
N/m. The ridge-push force magnitude is $\sim 1 - 1.5 \times 10^{12}$ N/m, which, again falls short
of providing the right magnitude of vertically integrated deviatoric stress for supporting
the Tibetan plateau. The large N-S deviatoric extension in Tibet calls for an additional
N-S compressional force of $\sim 3 \times 10^{12}$ N/m that can cancel the deviatoric extension.
The stress magnitudes for Tibet as well as the ridge-push force are a factor of two lower
that what was proposed in previous studies.

Figure DR1. Global distribution of vertically integrated horizontal deviatoric stresses with a uniform viscosity distribution ($\mu = 1$) for both plate boundaries and plate interiors. Topography is in meters. Besides the solution in Fig. DR7, all the other solutions are compensated.

Figure DR2. Relative viscosity distribution for all the plates. The white areas represent intra-plate regions with reference viscosity 1. The deforming areas are assigned viscosities inversely proportional to the strain rate. A reference viscosity of $\mu_{ref} = 0.01$ is chosen at the moderately spreading mid-oceanic ridge in the Indian Ocean. Places with higher viscosities than $\mu_{ref}$ are deforming at a slower rate.

Figure DR3. Global distribution of vertically integrated horizontal deviatoric stresses. The plate boundaries are assigned variable viscosities based on their strain rates. A reference viscosity, $\mu_{ref}$ of 0.01 is used for the mid-Indian ridges. This compensated solution corresponds to the uncompensated solution in Fig. DR7. Topography is in meters.

Figure DR4. Global distribution of vertically integrated horizontal deviatoric stresses. The plate boundaries are assigned variable viscosities based on their strain rates. A reference viscosity, $\mu_{ref}$ of 0.001 is used for the mid-Indian ridges. Topography is in meters.

Figure DR5. Relative viscosity distribution for all the plates. The white areas represent high viscosity intra-plate regions. The difference with Fig. DR2 is that Tibet and the Himalayas are assigned viscosities equal to the undeforming plate interiors.

Figure DR6. Global distribution of vertically integrated horizontal deviatoric stresses.
The plate boundaries are assigned variable viscosities based on their strain rates, except the Tibetan Plateau and the Himalayas which are assigned a viscosity equal to the undeformed plate interiors. A reference viscosity of 0.01 is used for the mid-Indian ridges. Topography is in meters.

Figure DR7. Global distribution of vertically integrated horizontal deviatoric stresses in the uncompensated case corresponding to the compensated case in Fig. DR3. The plate boundaries are assigned variable viscosities based on their strain rates. A reference viscosity, $\mu_{ref}$ of 0.01 is used for the mid-Indian ridges. Topography is in meters.
Figure DR2

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