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Magma reservoirs from the upper crust to the Moho inferred from high-resolution Vp and Vs models beneath Mount St. Helens, Washington State, USA
Eric Kiser et al.

SUPPLEMENTARY MATERIALS

Data

Travel times from first-arriving P (Pg) and S (Sg) waves and P wave Moho reflections (PmP) are inverted for P and S wave seismic velocities. Seismic data were recorded along two lines of instruments oriented N42E (X) and S57E (Y) with lengths of 152 km and 195 km, respectively (Figure 1 – main text). The X and Y lines included 937 and 1079 instruments and average station spacings of 160 and 180 m, respectively. Most instruments along these two lines were Reftek 125A (Texan) seismographs with 4.5 Hz geophones. Within 7.5 km of Mount St. Helens, Nodal Seismic instruments with 10 Hz geophones were also incorporated into the data sets. Eight borehole shots were recorded along each line of instruments. The inner four shots were ~450 kg and were approximately 15 and 30 km from Mount St. Helens. The outer shots were ~900 kg and were approximately 50 and 75 km from Mount St. Helens. In addition to the eight shots, an M 2.1 earthquake with a depth of 19.1 km and epicenter near the southwest end of the X line was recorded during the active-source experiment. Travel time data from this event are used in the X line inversion. Record sections with Pg, PmP, and Sg travel time picks from outer and inner shots are shown in Figure DR1. Additional reflections observed in the data are not used in this study. The data are band-pass filtered from 1-16 Hz for the Pg and PmP picks and 1-10 Hz for the Sg picks. The final data set used for the inversions consists of 11,525 Pg
picks, 1957 PmP picks, and 8952 Sg picks. Data uncertainties for Pg, Sg, and PmP are 0.075 s, 0.2 s, and 0.2 s, respectively.

**Inversion**

Pg and Sg travel times are inverted using the First Arrival Seismic Tomography (FAST) program package, an iterative, regularized, ray-based inversion (Zelt and Barton, 1998). Wavelength-dependent pre-smoothing is applied to velocity models as a computationally efficient way of approximating finite-frequency effects on travel times (Zelt and Chen, 2015). The dominant frequency of the data is 4 Hz, which is used for the pre-smoothing. A finite difference approximation is used to solve the eikonal equation on a uniform grid for travel times from each source (Vidale, 1988). Ray paths are defined by the steepest gradients interpolated from the gridded travel times from the receivers to the sources, and are recalculated at each iteration. The regularized inversion seeks to minimize an objective function that is the weighted combination of the L2 norm of data misfit and model smoothness. A least-squares conjugate gradient method is used to determine model updates. Iterations continue until the normalized chi-squared value is equal to one, or data misfit stops improving. Inversions typically require 10-20 iterations with 6 to 8 tradeoff values between data misfit and model smoothness tested during each iteration.

PmP travel times are also inverted using an iterative, ray-based program package (RAYINR) (Zelt and Smith, 1992). Here the models are parameterized as a series of layers with a varying number of nodes at the top and bottom of each layer defining seismic velocities and layer thickness. Rays are calculated by numerically solving the 2D ray tracing equations using a Runge-Kutta method. Model updates are determined using a damped least-squares inversion.
The coarse model parameterization reduces the sparseness of the matrix that is to be inverted, allowing for a LU decomposition to be used. As with Pg and Sg travel times, iterations continue until an acceptable data misfit is reached. Typical inversions require around 5-10 iterations with multiple damping parameter values tested at each iteration.

Final Vs models result from the iterative FAST inversion, using a smooth 1D velocity model as the starting model. Final Vp models result from three separate iterative inversions. Pg travel times are first inverted using FAST, starting with a smooth 1D model. The final models from this inversion are used as the starting models of the RAYINVR inversion of PmP travel times. In addition to velocity nodes, the starting RAYINVR models have a starting Moho boundary depth consistent with a reflection/refraction survey ~40 km north of Mount St. Helens (Parsons et al., 1999). The boundary and velocity nodes below 10 km depth are allowed to update during the inversion. In addition, velocity models from the two lines are constrained to be equal at the intersection nodes below 10 km. The final RAYINVR models are then used as the starting models for a final FAST inversion. This final inversion is used to smooth out velocity contrasts caused by holding the upper 10 km of the models constant during the RAYINVR inversion.

The grid spacing for all FAST inversions is 0.5 km. The node locations for the RAYINVR inversions vary with depth. For the upper 10 km of the model, velocity nodes are distributed as a uniform grid with 2 km spacing. Below 10 km, velocity nodes are present at 15, 20, 25, and 32 km depth. The lateral spacing of these nodes is 10 km. The Moho is defined by a series of co-located boundary and velocity nodes separated by 10 km laterally.

**Checkerboard Tests**
Model resolution is evaluated using checkerboard tests (Zelt, 1998). Positive and negative perturbations with maximum amplitudes of 10%, and the form of $\sin(x)\sin(z)$, are added to final Vp and Vs models. Synthetic Pg, Sg, and PmP travel time data, which are used as the observations, are generated from these checkerboard models. Gaussian noise with a standard deviation the same as the uncertainty of the real data picks is added to these travel time data. Checkerboard data are inverted using the same procedures as those used for the real data. For these inversions, the final Vp and Vs models are used as the starting models.

Comparisons between the recovered velocity perturbations and input checkerboards indicate the resolution in different parts of the model. Square checkerboard cells with dimensions of 5, 10, 15, 20, and 25 km are tested. In order to quantify similarities between input and recovered velocity perturbations, the semblance ($S_i$) within a 5 km circle around the $i$th grid point, which is equal to

$$S_i = \frac{\sum_{j=1}^{N_i} (PT_{ij} + PR_{ij})^2}{2 \sum_{j=1}^{N_i} (PT_{ij}^2 + PR_{ij}^2)},$$

is calculated. Here $N_i$ is the number of grid points within a 5 km circle centered on the $i$th grid point, and $PT_{ij}$ and $PR_{ij}$ are the true and recovered velocity perturbations, respectively, at the $j$th grid point in the 5 km circle surrounding the $i$th grid point. For each cell size, three models are tested that shift the checkerboard vertically and horizontally by the half-cell width to evaluate the effects of the positioning and polarity of the velocity perturbations. The average semblance of the three models is used to evaluate model resolution (Figures DR2-DR5). Assuming that semblance values of 0.7 or higher indicate well-resolved model parameters (Zelt, 1998), Figures DR2 and DR4 indicate that in general Vp perturbations with dimensions of 10 km can be resolved down to 5–10 km depth, perturbations with dimensions of 15 km can be resolved to 25-30 km depth, and perturbations with dimensions of 20 km or larger can be resolved throughout
most of the crust. Vs perturbations with dimensions of 10km can be resolved down to ~5 km depth (Figures DR3B and DR5B). Perturbations with dimensions of 20-25 km can be resolved down to at least 15 km beneath Mount St. Helens (Figures DR3D-E and DR5D-E).

**Model uncertainties**

In order to evaluate the robustness of the Vp/Vs anomalies presented in the main text, uncertainties have been estimated for Vp (Figure DR6), Vs (Figure DR7), and Vp/Vs (Figure DR8) in the top ~15 km of the models. These uncertainties have been estimated using the jackknife method for Vp and Vs along both lines (e.g., Lees and Crosson, 1989; White and Clowes, 1990). For each velocity and line, we use 30 subsets of the data (e.g., 30 subsets of the P picks along the X line). The use of 30 subsets results in the removal of approximately 190 and 150 picks of Pg and Sg, respectively, for each subset. For each line, estimated uncertainties in Vp and Vs are propagated to estimate the uncertainties in Vp/Vs.

Vp uncertainties are below 0.05 km/s along both lines (Figure DR6). Vs uncertainties are relatively high along the Y line, with values above 0.2 km/s for much of the velocity model (Figure DR7B). Uncertainties in Vp/Vs are typically below 0.1 along the X line (Figure DR8A). A significant portion of the Y line has Vp/Vs uncertainties above 0.1 (Figure DR8B). The analysis suggests that the high Vp/Vs anomaly beneath Mount St. Helens is a robust feature. The high Vp/Vs anomaly beneath Indian Heaven Volcanic Field (IH) has a higher uncertainty, particularly near the upper portion of this feature. It should be noted that the large uncertainties beneath IH come from variations in the internal structure of the F4 anomaly, not the presence or absence of this feature.
Travel time misfit

Variance reductions are calculated by comparing predicted travel times from the final velocity models with those predicted from average 1D velocity models. For the X line, variance reductions of 93%, 88%, and 64% are achieved for Pg, Sg, and PmP, respectively. Variance reductions of 97%, 93%, and 83% are achieved for Pg, Sg, and PmP, respectively, for the Y line. The final velocity models have normalized chi-squared values of 0.95, 1.09, and 1.75 for Pg, Sg, and PmP along the X line, and 1.17, 1.51, and 1.42 for Pg, Sg, and PmP along the Y line. RMS travel time residuals for Pg, Sg, and PmP are 0.073s, 0.21s, and 0.26s along the X line, and 0.081s, 0.25s, and 0.24s along the Y line.

Additional Models

Additional models include the final Vs models (Figure DR9), the Vs velocity perturbations from the initial smooth 1D model (Figure DR10), the final Vp models plotted with recent seismicity (Figure DR11), the Vp velocity perturbations from the initial smooth 1D model (Figure DR12), and the Vp/Vs models plotted with recent seismicity between 2009 and 2015 (Figure DR13).

For the models presented in the main text a small uncertainty of 0.1 km is used for the Moho location nodes so as to maintain a relatively smooth Moho. Increasing this uncertainty produces a model with more abrupt and larger amplitude changes in the Moho interface and smaller lower crustal velocity anomalies. The main problem with an irregular Moho is that it increases the number of observations for which rays fail to be traced between the shots and stations with the RAYINVR program. Tracing rays between shots and receivers is a fundamental constraint on any model, so we prefer models that maximize the number of rays that
can be traced while maintaining acceptable data misfits.

For completeness, we have included a model with irregular Moho features for which we have increased the uncertainty of Moho location nodes to 0.25 km (Figure DR14). Using this parameter value, 50 fewer rays are traced compared with the models in the main text. The amplitudes of the lower crustal velocity anomalies have been reduced, but the general pattern of the velocity anomalies remains the same, hence so do our interpretations. Note that above ~30 km depth, velocities are very similar to those in the original model.

**Preliminary 3-D P wave picks**

We have plotted the residuals of P wave travel time picks for the 3D data set for those shots for which this process has been completed (Figure DR15). The residuals are calculated based upon the difference between observed travel times and calculated travel times using a 1D velocity model produced by averaging velocities of the 2D lines shown in the main text (Figure 2). In order to remove the effect of shallow structure near each shot, we have also subtracted half the mean travel time residuals from stations within 5 km of each shot. Figure DR15 shows that similar travel time residuals tend to occur over length scales of 10s of kilometers, indicating that 2D travel time tomography is a valid approach in this region.

**SUPPLEMENTARY REFERENCES CITED**


Parsons, T., Wells, R.E., Fisher, M. a., Flueh, E., and ten Brink, U.S., 1999, Three-dimensional velocity structure of Siletzia and other accreted terranes in the Cascadia forearc of


Figure DR1: Record sections from a 900 kg shot recorded along the X line (A and B) and a 450 kg (C and D) shot recorded along the Y line. The 900 and 450 kg shots are located at 122.88 W, 45.69 N and 122.57 W, 46.35 N, respectively. Red, green, and purple dots show the Pg, PmP, and Sg travel time picks. All data are reduced using a velocity of 7 km/s and band-pass filtered between 1 and 10 Hz.
Figure DR2: Semblance between input and recovered checkerboard model perturbations on the final X line Vp model with dimensions of 5 (A), 10 (B), 15 (C), 20 (D), and 25 km (E). Semblance values are the average of three checkerboard tests that evaluated the effect of 5 km vertical and horizontal shifts in the checkerboard perturbations. The contour interval is 0.1, and the thick white line is the 0.7 semblance value that indicates well-resolved model parameters.

Figure DR3: The same as Figure DR2 except for Vs perturbations.
Figure DR4: The same as Figure DR2 except for the Y line.
Figure DR5: The same as Figure DR3 except for the Y line.
Figure DR6: Vp standard deviations for the X (A) and Y (B) lines.

Figure DR7: Vs standard deviations for the X (A) and Y (B) lines.
Figure DR8: Vp/Vs standard deviations for the X (A) and Y (B) lines.

Figure DR9: Final Vs models for the X (A) and Y (B) lines.
Figure DR10: Percentage Vs change from the initial smooth 1D model for the X (A) and Y (B) lines.
Figure DR11: Final Vp models for the X (A) and Y (B) lines. Black dots are earthquake locations between January 2009 and April 2015. Teal dots are all recorded DLP events.
Figure DR12: Percentage Vp change from the initial smooth 1D model for the X (A) and Y (B) lines.
Figure DR13: Vp/Vs for the X (A) and Y (B) lines. White dots are earthquake locations between January 2009 and April 2015.
Figure DR14: Vp for the X (A) and Y (B) lines using 0.25 km for the Moho boundary node uncertainty.
Figure DR15: Preliminary 3D P wave pick residuals. White stars are shot locations and dots are seismograph locations. The colors of the dots represent the residuals of the observed P wave picks minus calculated travel times using a 1D model.