Deriving rock uplift histories from data-driven inversion of river profiles

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DATA REPOSITORY

DR1: Methodological details
The rate of change in elevation of the river profile depends on the rock uplift \( U \) and erosion rate \( E \) (e.g., Whipple and Tucker, 1999), and thus the evolution equation can be written as:

\[
\frac{\partial z}{\partial t} = U_{x,t} + E_{x,t}
\]  

(1)

The lower cases in equation (1) indicate that the uplift and erosion rate can vary with time (\( t \)) and location (\( x \)). Erosion counteracts uplift and degrades uplifted river profiles. Based on geomorphological investigations, fluvial erosion ‘laws’ have been implemented, such as the transport-limited or detachment-limited model (e.g., Whipple and Tucker, 1999). Here the so-called stream power model is used, which assumes that the rate of incision is primarily controlled by discharge and channel slope, which is used as a proxy for stream power (e.g. Whipple and Tucker 1999). The erosional coefficient (\( K \)) is laterally and temporally constant in the presented applications, but the program also allows to vary \( K \) laterally. Temporal variations of discharge and \( K \) induced by orbital cycles have little effect on the presented river profile modelling, because the length and timescales of the model is much larger than the climate variations (e.g. Paul et al. 2014). The concavity (\( m_r/n_r \)) and exponent of the river gradient (\( n_r \)) are free parameters in all models and allowed to vary from 0.4-0.7 and from 2/3 to 5/3, respectively (e.g., Whipple, 2001). All models start with flat river profiles, which are set to the present-day outlet elevation of individual trunk streams. The latter is invariant during modelling and used as model boundary condition.

The presented model enables to take into account the flexural isostatic uplift as a result of changes in topography. The isostatic uplift is modelled by solving the bi-harmonic equation similar to the approach in PECUBE (Braun et al., 2012):

\[
D \frac{\partial^4 \Delta u}{\partial x^4} + D \frac{\partial^4 \Delta u}{\partial y^4} + 2D \frac{\partial^2 \Delta u}{\partial x^2 \partial y^2} = \Delta \rho g \Delta u + \rho_0 g \Delta e
\]  

(2)

where \( D \) is the lateral constant flexural rigidity given by:

\[
D = \frac{ET_e^3}{12(1 - \nu^2)}
\]

\( T_e \) is the equivalent elastic thickness of the plate, \( E \) is the Young’s modulus, \( \nu \) is the Poisson’s ratio, \( g \) is the acceleration due to gravity and \( \Delta e \) and \( \Delta u \) are the increment in
surface topography and isostatic uplift over a time step $\Delta t$ of computation. An efficient Fourier-transform approach was used to solve the so-called ‘thin elastic plate equation’ (Nunn and Aires, 1988). The isostatic uplift is added to the input rock uplift and thus either increase or decrease the overall rock uplift. The complex interaction between the transient topographic evolution and the input uplift history result in complex uplift paths that show distinct differences compared to the ‘step-like’ uplift paths modelled without isostasy (cf. Fig. DR3D and Fig. DR4D).

The misfit of modelled and observed data is regulating the inversions, and is calculated with the log-likelihood ($\ln L$) function for Gaussian-distributed errors:

$$
\ln L = -\sum_{i=1}^{n} \ln(2\pi) + \ln \sigma_i + 0.5 \left( \frac{o_i - p_i}{\sigma_i} \right)^2
$$

where $p_i$ is the predicted value (elevation, thermochronological age, nuclide concentration), and $o_i$ and $\sigma_i$ is the observed value and 1σ error of observation $i$ and $n$ is the total number of observations. The $\ln L$ of each observation class (elevation of river nodes, cosmogenic nuclide concentrations, thermochronological ages and fission track length distributions) is normalised (divided) by the number of observations and the sum of normalised $\ln L$s regulates the inversion, avoiding preferential fitting of the largest dataset. Analytical uncertainties of thermochronological and cosmogenic nuclide data are well known, ranging from approximately 5 to 20%. As an example the mean errors of the apatite fission fission track ages and $^{10}$Be concentrations from the Sila massif are 10 and 6%, respectively. The vertical absolute height error is less than 16 m for 90% of the global shuttle radar topography mission (SRTM) data (e.g. Rabus et al. 2003), but river profiles extracted from steeply incised streams will be associated with much larger errors. I used an arbitrary height error of 75 m, which corresponds to a mean relative error of 10%.

The neighbourhood algorithm inversion was used to efficiently explore the multi-dimensional parameter space and extract statistical parameters (Sambridge 1999a,b). This study used the parameterisation suggested by Glotzbach et al. (2011). Three main parameters control the convergence behaviour of the inversion: (i) the number of iterations ($N_i$), (ii) the number of models generated at each iteration ($N_m$) and (iii) the number of Voronoi cells re-sampled at each iteration ($N_r$). In this study $N_i$ is 200 (inversion test) and 400 (Sila massif), $N_m$ is 100 and $N_r$ is 90. The rather high resampling ratio (0.9) led to a rather slow convergence and ensures that the full parameter space is explored.

Bayesian estimates such as 1D marginal PDFs are first derived separately for each dataset and finally they are combined with:

$$
P(C|D_1,D_2,D_3) = \frac{\prod_{s=1}^{3} P(C|D_s)}{P(C)}
$$

where $P(C)$ is the prior probability of parameter $C$ and $P(C|D_s)$ are the posterior probabilities of independent datasets, here the river profile, the cosmogenic and thermochronological data.

River long profiles were extracted from shuttle radar topography mission (SRTM) digital elevation models (DEM) with a vertical accuracy of ±16 m. Only those stream
segments with an accumulation area >1 km$^2$ were extracted from the DEM, and smoothed with a moving average window of 1 km. The resulting river profile is resampled to 1 km distance between successive river nodes. The Sila massif model consists of 129 river nodes, and each river node is connected to a 500 m long hillslope discretised into 10 nodes (50 m spacing).

The computation time of a single forward model of the Sila massif is 16.4 CPU seconds (on a 2.66 GHz Quad-Core Intel Xeon), whereas the solution of the river-hillslope model takes 7 seconds, the calculation of $^{10}$Be concentration at all river and hillslope nodes takes 8 seconds and the 1D thermal modelling takes 1.4 seconds. Increasing the number of river nodes increases only slightly the computational time of the river-hillslope model (Fig. DR1); the calculation of 261 river nodes takes 10 seconds compared to 7 seconds for 129 river nodes.

Fig. DR1: Solution time of a the river-hillslope model for a single forward model as a function of the number of river nodes.
DR2: Testing the performance of the model

Two synthetic datasets (river profile, cosmogenic and thermochronological data) were generated with 1.) increasing uplift rates and 2.) decreasing uplift rates. In both examples, river nodes were extracted from the Acher catchment in the northern Black Forest in southwestern Germany.

1.) The synthetic data is modelled with an uplift rate of 0.1 km/Ma from 50 to 1 Ma, followed by 0.5 km/Ma uplift until present. The river profile was modeled assuming that river incision is proportional to stream power and the erosional coefficient ($K$) and exponents of discharge ($m$) and slope ($n$) are $1 \times 10^{-6}$ yr$^{-1}$, 0.5 and 1, respectively. The resulting river profile is characterised by a downstream increase in steepness with a distinct knickzone at 6-10 km upstream of the bounding node (Fig. DR2A). Recorded erosion rates and cooling paths were used to calculate at all model nodes (63 river nodes and 630 hillslope nodes) the concentration of cosmogenic $^{10}$Be and at three locations apatite (U-Th)/He ages and apatite fission track ages with corresponding length distributions (Fig. DR2A-C and DR3A-C). The synthetically generated analytical data was used for inverse modeling of seven parameters: P1) the time of change in uplift rate, P2-3) the uplift rate before and after P1, P4) the erosional coefficient ($K$), P5) the exponent of slope ($n$), P6) the concavity index ($m/n$) and P7) the geothermal gradient. Convergence of the likelihood function is achieved after running a few thousands models in a few hours. Inversion results obtained by inverting only the river profile without analytical data reveal that single parameters, such as the exponent $n$ and the concavity index ($m/n$) can be successfully retrieved (Fig. DR2D). The complete uplift history, especially the timing of the increase in uplift rate, however, can only be reconstructed by the integrated inversion of all datasets including the thermochronological and cosmogenic nuclide data (Fig. DR2D).
Fig. DR2: Results of inverse modeled synthetic river profile, thermochronological and cosmogenic \(^{10}\)Be data. Synthetic data was modeled with an erosional coefficient \((K)\) and exponents of discharge \((m)\) and slope \((n)\) of \(1\times10^{-6}\) yr\(^{-1}\), 0.5 and 1, and 0.1 km/Ma uplift from 50 to 1 Ma, followed by 0.5 km/Ma until present. A) Synthetic river profile, apatite fission track age and length distribution, apatite (U-Th)/He age and cosmogenic \(^{10}\)Be nuclide concentrations of sample localities. The best-fit modeled data is shown in comparison to the synthetic data. B) Erosion rate evolution of modeled river nodes of the trunk stream for the last 1.2 Ma. C) Modeled cooling curves of three sample localities along the trunk stream used to calculate thermochronological data. D) 1D marginal distributions of 20000 forward models with their mean value and standard deviations. The black distributions are derived from an inversion based on all data (river profile plus analytical data) and the red distributions are from an inversion based solely on the river profile.

2.) The synthetic data is modelled with an uplift rate of 1.0 km/Ma from 20 to 1 Ma, followed by 0.1 km/Ma uplift until present. The river profile was modeled assuming an erosional coefficient \((K)\) of \(1\times10^{-7}\) yr\(^{-1}\) and exponents of discharge \((m)\) slope \((n)\) of 0.5 and 1.25, respectively. The resulting river profile is characterised by an upstream increase in steepness (Fig. DR3A). Erosion rates and cooling paths were extracted from similar sample locations as in first example and used to calculate the concentration of cosmogenic \(^{10}\)Be and apatite (U-Th)/He ages and apatite fission track ages with corresponding length distributions (Fig. DR3A-C). The synthetically generated analytical data was used for inverse modeling of seven parameters: P1) the time of change in uplift rate, P2-3) the uplift rate before and after P1, P4) the erosional coefficient \((K)\), P5) the exponent of slope \((n)\), P6) the concavity index \((m/n)\) and P7) the geothermal gradient. Convergence of the likelihood function is achieved after running a few thousands models. Extracted 1D marginal distributions of the
posterior probability distribution predict most of the free parameter correctly, except for the erosional coefficient (k) and the exponent of slope (n). The uplift history, however, is correctly retrieved by the inverse modelling (Fig. DR3D).

Fig. DR3: Results of inverse modeled synthetic river profile, thermochronological and cosmogenic $^{10}$Be data. Synthetic data was modeled with an erosional coefficient (K) and exponents of discharge (m) and slope (n) of $1 \times 10^{-7}$ yr$^{-1}$, 0.5 and 1.25, and 1.0 km/Ma uplift from 20 to 1 Ma, followed by 0.1 km/Ma until present. A) Synthetic river profile, apatite fission track age and length distribution, apatite (U-Th)He age and cosmogenic $^{10}$Be nuclide concentrations of sample localities. The best-fit modeled data is shown in comparison to the synthetic data. B) Erosion rate evolution of modeled river nodes of the trunk stream for the last 1.2 Ma. C) Modeled cooling curves of three sample localities along the trunk stream used to calculate thermochronological data. D) 1D marginal distributions of 20000 forward models with their mean value and standard deviations.
Fig. DR4: Results of inverse modelled Trionto river profile, thermochronological and cosmogenic $^{10}$Be data, taking into account uplift caused by flexural isostasy with the following parameters: elastic thickness of 5 km, density of the crust and mantle 2700 and 3200 kg/m$^3$, Young’s modulus $1 \times 10^{11}$ Pa, Poisson ratio of 0.25. A) Observed and best-fit river profile, apatite fission track age, mean apatite fission track length (MTL) and cosmogenic $^{10}$Be nuclide concentrations of sample localities. The observed river profile is color coded according to lithology (green: metamorphic rocks, blue: Mesozoic-Eocene sediments, red: granite). The best-fit model yield a normalized misfit of 19 for the following parameters (P1-P9): 16.3 Ma, 0.86 Ma, 1.90 mm/a, 0.07 mm/a, 1.45 mm/a, $4.12 \times 10^{-6}$, 0.69, 0.63, 21.3 °C/km. B) 1D marginal distributions of the posterior probability distribution with their mean values and standard deviations. The lower right diagram shows the 2D marginal distribution of the uplift history derived by combining parameters P1-P5. The thick black and red line are the total rock uplift (input uplift plus uplift caused by changes in topography) for the best-fit model for the lowest (red) and higher (black) river node. C) 2D marginal distributions of the posterior probability distribution of parameter pairs that show obvious trade-offs.
**DR4: River profile evolution of the Trionto river, Sila massif, Italy**

Fig. DR5: Modeled river profile at 20, 15, 10, 5, 1, 0.5, 0 and in 2 Ma of the Trionto river for the best-fit model with following parameters: (i) an erosional coefficient ($k$) of $4.36 \times 10^{-6}$, (ii) an exponent $n$ of 1.25, (iii) a concavity index ($m/n$) of 0.40, (iv) a geothermal gradient of 25.5 °C/km and (v) an initial uplift of 1.5 km/Ma until 17.5 Ma, followed by 0.08 km/Ma uplift until 0.9 Ma and finally 0.78 km/Ma uplift.

**References cited in the Data Repository**


Sambridge, M., 1999b, Geophysical inversion with a neighbourhood algorithm — II.