Supplementary Material for Rheological controls on the emplacement of extremely high-grade ignimbrites

A.1. Details of Numerical Modeling

The time-dependent conduction of heat in one dimension considering strain heating as the only internal heat production can be written as:

\[
\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial z^2} + \frac{Q_{\text{strain}}}{C_p \rho}
\]  

(A1)

where \( T \) is temperature, \( t \) is time, \( D \) is the thermal diffusivity, \( z \) is the depth in the shear zone, \( C_p \) is the heat capacity of the melt, and \( \rho \) is the density of the melt. \( Q_{\text{strain}} \) is the volumetric strain heating in the shear zone, and is given by:

\[
Q_{\text{strain}} = \sigma \dot{\varepsilon}
\]  

(A2)

Here \( \sigma \) is the shear stress, and \( \dot{\varepsilon} \) is the strain rate. Assuming the melt in the shear zone is behaving as a Newtonian fluid with viscosity \( \eta \), we can define shear stress as:

\[
\sigma = \eta \dot{\varepsilon}
\]  

(A3)

Substituting equations A3 and A2 into A1, we obtain:

\[
\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial z^2} + \frac{\eta \dot{\varepsilon}^2}{C_p \rho}
\]  

(A4)

\( D, C_p, \) and \( \rho \) are constants, and we can define a constant \( N \) equal to:

\[
N = \frac{k^2}{C_p \rho}
\]  

(A5)

and simplify equation A4 to:

\[
\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial z^2} + N \eta(T)
\]  

(A6)

We use the explicit finite difference method to solve equation A6 for strain heating. In one dimension, we define \( z \) nodal points and calculate the difference in temperature around the node
of interest (indicated with subscript $m$) for each time step $\Delta t$. Superscript $p$ designates each iterative step:

$$\frac{t_{m+1}^p - t_m^p}{\Delta t} = D \frac{(t_{m+1}^p + t_{m-1}^p - 2t_m^p)}{(\Delta z)^2} + N\eta(T)$$

(A7)

Equation A7 may be simplified to:

$$t_m^{p+1} = Fo \left( t_{m+1}^p - t_{m-1}^p \right) + (1 - 2Fo) t_m^p + N\eta(T) \Delta t$$

(A8)

Using the explicit method, the solution to equation A8 will be stable when the Fourier number $Fo \leq 0.5$, where $Fo$ is defined as:

$$Fo = \frac{D\Delta t}{(\Delta z)^2}$$

(A9)
<table>
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<th>Sample</th>
<th>$T$ °C</th>
<th>$\log_{10}\eta_{app*}$ log (Pa s)</th>
<th>$\varepsilon_{total}$ minutes</th>
<th>$\rho_{total}$ pre kg m$^{-3}$</th>
<th>$\rho_{total}$ post kg m$^{-3}$</th>
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</table>

* Newtonian behavior was observed over the range of temperatures and strain rates ($10^{-6}$ - $10^{-8}$ s$^{-1}$) investigated.