1 Henry’s Law

Henry’s law calculation for outgassing of pressurized water during subglacial earthquake.

Consider a cavity of initial volume \( V \), completely filled with water saturated with nitrogen, that expands by

\[
dV = dV_l + dV_g
\]

during an earthquake, with the subscripts indicating liquid and exsolved gas.

From Henry’s Law, the volumetric concentration of nitrogen molecules in the water is \( C = P/K \) with \( P \) the pressure and \( K \) the Henry’s-law constant ([Mackay and Shiu(1981)]). The number of nitrogen molecules in the water is then \( N = PV/K \). Noting that \( dV_l << V \), the number of molecules exsolving into a gas phase during the quake, \( N_g \), is the difference between those in the nearly-unchanged volume of water \( V \) at the initial pressure \( P_1 \) and the final pressure \( P_2 \),

\[
N_g = V(P_1 - P_2)/K
\]

Then, using the gas law at absolute temperature \( T \) with gas constant \( R \),

\[
dV_g = N_g RT/P_2
\]

The water has compressibility \( \beta \), so

\[
dV_l = \beta V(P_1 - P_2)
\]

Substituting into equation 1 from equations 3 and 4 yields

\[
dV = \beta V(P_1 - P_2) + N_g RT/P_2
\]

and substituting for \( N_g \) from Equation 2

\[
dV = \beta V(P_1 - P_2) + \frac{V(P_1 - P_2)RT}{KP_2}
\]

Multiplying by \( P_2 \) and dividing by \( V \) and \( \beta \) gives

\[
\frac{P_2dV}{\beta V} = -P_2^2 + P_1P_2 - P_2 \left( \frac{RT}{\beta K} \right) + P_1 \left( \frac{RT}{\beta K} \right)
\]

Then, adding or subtracting to move everything to the left-hand side, and grouping in terms of \( P_2 \) gives

\[
P_2^2 + P_2 \left( \frac{dV}{\beta V} - P_1 + \frac{RT}{\beta K} \right) - \frac{RT}{\beta K} P_1 = 0.
\]

which is the quadratic for the final pressure \( P_2 \). The solution with physically possible, positive pressure is
\[ P_2 = \frac{-\left( \frac{dV}{\beta V} - P_1 + \frac{RT}{\beta K} \right) + \left[ \left( \frac{dV}{\beta V} - P_1 + \frac{RT}{\beta K} \right)^2 + \frac{4RTP_1}{\beta K} \right]^{0.5}}{2} \]  

(9)

The constants are \( K = 10^5 \text{ m}^3 \text{ Pa}^{-1} \text{ mol}^{-1} \) ([Rettich et al. (1984) Rettich, Battino, and Wilhelm]), \( \beta = 5 \times 10^{-10} \text{ Pa}^{-1} \) ([Fine and Millero (1973)]), and \( R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1} \). Temperature will be close to or slightly below \( T = 273 \text{ K} \), depending on pressure.

For air-free water, our specified strain of \( dV/V = 5 \times 10^{-3} \) would drop the pressure from 10 MPa to 0, sufficient to cause cavitation beneath about 1100 m of ice, but the pressure would drop only to 8 MPa if complete equilibrium were achieved starting from water fully saturated with nitrogen.

References

