Methods

To provide a minimal model for melt flow at Sapat, we modify the formulation of Connolly and Podladchikov (2007) for fluid flow through a porous matrix with differential viscous yielding to account for temperature dependent solubility of the matrix. Assuming the small porosity limit \(1 - \phi \to 1\), isostatic compaction, the fluid and solid are incompressible and have the same specific heat, and viscous dissipation is insignificant, the governing equations are conservation of solid mass

\[
\frac{\partial \phi}{\partial t} = -\frac{p}{\zeta} - k \left( \nabla p_e + \Delta \rho \bar{g} u_z \right) \left( \frac{\partial c_T}{\partial T} \nabla T \right)^T,
\]

conservation of total mass combined with Darcy’s law

\[
\frac{p_e}{\zeta} = \frac{1}{\mu_f} \nabla \cdot \left( k \left( \nabla p_e + \Delta \rho \bar{g} u_z \right) \right) + k \left( \nabla p_e + \Delta \rho \bar{g} u_z \right) \left( \frac{\partial c_T}{\partial T} \nabla T \right)^T \left( 1 - \frac{\rho_s}{\rho_f} \right),
\]

and conservation of energy

\[
\frac{\partial T}{\partial t} = \kappa \nabla^2 T + \nabla \cdot \left( \frac{k}{\mu_f} \left( \nabla p_e + \Delta \rho \bar{g} u_z \right) T \right) + h \frac{k}{\mu_f} \left( \nabla p_e + \Delta \rho \bar{g} u_z \right) \left( \frac{\partial c_T}{\partial T} \nabla T \right)^T,
\]

where \(\phi, t, T, p_e\) and \(c\) are, respectively, porosity, time, temperature, effective pressure, and the volumetric solubility of the solid in the fluid; \(g\) is gravitational acceleration; \(u_z\) is an upward directed unit vector; the fluid shear viscosity \((\mu_f)\), thermal diffusivity \((\kappa)\), densities \((\rho_s, \rho_f, \Delta \rho = \rho_s - \rho_f)\), and latent heat of solution \((h)\) normalized by specific heat are constants. The permeability \((k)\) and bulk viscosity \((\zeta)\) are given by the constitutive relations

\[
k = k_0 \left( \frac{\phi}{\phi_0} \right)^3
\]

and

\[
\zeta = \frac{\mu_s}{\phi} \left( R - H(p_e)(R - 1) \right)
\]

where \(k_0\) and \(\phi_0\) are the background values of the permeability and porosity, \(\mu_s\) is the shear viscosity of the solid, \(H\) is the Heaviside function, and \(R\) is the ratio of bulk
viscosity in decomaption to that in compaction. Convenient scales for non-dimensionalization are the viscous compaction length

\[ \delta = \sqrt{\frac{k_0 \mu_s}{\phi_0 / \mu_f}} \]

the characteristic pressure

\[ p^* = \delta \Delta p \]

the viscous compaction time scale

\[ \tau = \mu_s / \left( \phi_0 \delta \Delta p \right) \]

and the temperature difference between the top and base of the model domain

\[ \Delta T = T_{up} - T_{low} \]

Making use of these scales and the constitutive relations, the non-dimensional forms of the governing equations are

\[ \frac{\partial f}{\partial t'} = f \frac{p}{1 - H(p)(1 - R)} + f^3 \left( \nabla p - u_z \right) \left( \frac{\partial c}{\partial T'} \nabla T' \right)^T \]

\[ f \frac{p}{1 - H(p)(1 - R)} = \nabla \cdot \left( f^3 \left( \nabla p - u_z \right) \right) + f^3 \left( \nabla p - u_z \right) \left( \frac{\partial c}{\partial T'} \nabla T' \right)^T \left( 1 - \frac{\rho_{solid}}{\rho_{fluid}} \right) \]

and

\[ \phi_0 \frac{\partial T'}{\partial t'} = \frac{\kappa T}{\delta^2} \nabla^2 T' + \nabla \cdot \left( f^3 \left( \nabla p - u_z \right) T' \right) + \frac{h_0}{\Delta T} f^3 \left( \nabla p - u_z \right) \left( \frac{\partial c}{\partial T'} \nabla T' \right)^T \]

where \( f = \phi / \phi_0 \) is the relative porosity, \( p = -p_e / p^* \) is the dimensionless fluid overpressure relative to lithostatic conditions, and \( t' = t/\tau \) is the dimensionless time.

Our formulation is for the reversible solubility of a saturated phase or phase assemblage and strictly cannot describe the independent variation of orthopyroxene and clinopyroxene observed at Sapat if these variations are due to solubility effects. As an ad-hoc model for these variations we discriminate precipitated and dissolved solid, the former being attributed to orthopyroxene and the latter being attributed to clinopyroxene. The initial temperature distribution is taken to be the steady state temperature for non-
reactive melt flow through a matrix with uniform porosity. To induce two-dimensional flow instability at the onset of the model we introduce a radially symmetric Gaussian perturbation to the initially uniform porosity ($\phi_0$) near the base of the spatial domain. The perturbation has a maximum amplitude of $2\phi_0$ and a half-width radius of $0.7\delta$. To stabilize the numerical calculations, melt flow is non-reactive within $2\delta$ of the upper and lower boundaries. The numerical calculations are done by finite difference methods as described by Connolly and Podladchikov (2007) except that upwind finite differences are used to evaluate the advective term in the heat flow equation. Three-dimensional effects would be likely to enhance melt focusing, thereby enhancing the advective heat effects illustrated here.

The length and width of the model domain are $L = 50\delta$ and $W = 10\delta$. The compaction equations together with Eq A1 are fully parameterized by $\delta$, $\tau$, $\phi_0$, $T_{\text{up}}$, $T_{\text{low}}$, $\partial c / \partial T$, $h_0$, $\kappa$, and $R$. The temperature of dependence of the matrix solubility $\partial c / \partial T$ is taken to be 0.001 K$^{-1}$, i.e., the matrix melts over an interval of 1000 K. The enthalpy of melting $h_0$, estimated from the enthalpy and heat capacities of pyroxenes and olivine (Ghiorso et al., 2002), was taken to be 400 K; and $\kappa$ is taken to be $10^{-6}$ m$^2$/s. To the extent that the previous parameters can be considered to be constants, the absolute temperatures for the model are irrelevant, accordingly we take $T_{\text{up}} - T_{\text{low}} = L 0.007$ K/m. The final parameter, $R$, characterizes the mechanical compaction instability and is expected to be comparable to ratio of the width of channelized flow features to the distance between these features (Connolly & Podladchikov 2007). On the basis of the size and spacing of the Sapat dunites a lower bound on $R$ would be ~0.1 if the mechanism was indeed responsible for the channelized flow. Smaller values of $R$ would lead to stronger channelization and larger advective heat effects than those illustrated by our numerical models.

References:


Ghiorso, M. S., M. M. Hirschmann, P. W. Reiners, and V. C. Kress (2002), pMELTS: A revision of MELTS for improved calculation of phase relations and major element partitioning related to partial melting of the mantle to 3 GPa, Geochemistry Geophysics Geosystems, 3,
Map of the main part of the largest PEZ found in Sapat from which Fig. 2 is a zoom
Areas left “white” represent snow or gravels.

- Meta-gabbro and its Cpx rim
- Dunite
  - (a) massively Cpx impregnated
  - (b) Cpx impregnation trails
- Pl-rich segregate
  *Dated in Bouilhol et al. 2011*