Appendix DR1: Methods

Properties of 200 eddies were measured from still photographs and individual video frames of the B2 eruption phase, and 194 were measured from the B3 phase. The surfacial velocity field was analyzed using digitized video of the eruption and the Image-J plug-in Flow-J.\(^{18}\) Flow-J measures the velocity field of a stack of images by tracking changes in pixel intensity (brightness) from one image to the next (Abramoff et al., 2000). Still photos taken throughout the B2 and B3 phases indicate that eddy size and column appearance did not significantly vary throughout either phase. Radar observations indicate that column height grew during the B2 phase (from 15 to 18 km), and remained relatively steady through the B3 phase (~14 km) (Carey et al., 1990). After digitizing VHS video (30 frames per second), we selected 5 representative sequences from the B2 phase and 5 sequences from the B3 phase, with durations of 6-9 seconds each. Within each sequence, individual frames were sampled at intervals of 1/5 or 1/6 second, saved as an image stack in Image-J, and processed with the Uras algorithm within Flow-J. All images were scaled using the 1.9 km diameter crater rim. Based upon pixel size and uncertainties in measuring the crater rim, we estimate errors of <50 m and ± 10 m/s in measurements of eddy size and velocity, respectively. Integral length scales of the turbulent flow fields were calculated using two-point autocorrelations of the velocity fields (Bernard and Wallace, 2002). Supplemental Figure DR1 shows representative autocorrelations of the B2 and B3 column margins; in these particular plots, the integral length scale of the B3 phase is ~250 m, whereas that of the B2 phase is >1000 m.

The gradient Richardson number, $Ri$, is calculated as
where \( g \) is gravitational acceleration, \( \rho \) is fluid density, \( r \) is radial distance, and \( u \) is velocity. \( Ri \) provides a measure of instability (and mixing) along a shear boundary such that \( Ri \leq 0.25 \) indicates mixing across the boundary, whereas greater values of \( Ri \) indicate little or no mixing.

Appendix 2: Grain size distributions

Although the grain size distributions of Plinian fall deposits from each eruption phase are different, only two initial, or eruptive, distributions are required to reproduce the observed variation (Supplemental Figure DR2). During the B1 phase, a very lithic-rich distribution was erupted, whereas a lithic-poor distribution was erupted during the B2, B3, and B4 phases. Initial grain size distributions, \( GSD_o \), are calculated from Plinian fall deposit distributions using the expression

\[
GSD_o = \sum_{\phi} \frac{S_{\phi o}}{v_{\phi}}
\]

Equation DR2

where particles, \( \phi \), vary in size and componentry, \( v_{\phi} \) is the fall velocity specific to each component and size, and \( S_{\phi o} \) is the initial sedimentation rate. The initial sedimentation rate is calculated, following (Sparks et al., 1992), as

\[
S_{\phi o} = \frac{S_{\phi}}{exp\left(\frac{-\pi(r^2 - r_o^2)v_{\phi}}{Q}\right)}
\]

Equation DR3

where \( S_{\phi} \) is the weight fraction of each grain size and component in a Plinian fall deposit, \( r \) and \( r_o \) are the radial distances of the sample location and plume corner from the vent, respectively, and \( Q \) is the buoyant volumetric eruption rate.
Locations 05MSH-12 and 05MSH-16 are indicated with the black square and circle, respectively, on the map in Supplemental Figure DR2. Lithic isopleths (in mm) of the B2 and B4 phases are indicated with solid and dashed lines, respectively (after Carey et al., 1990).

Figure DR1:
Figure DR2:

Grain Size Distributions

Natural Plinian Fall Samples

Calculated Initial (Bruptive) Size Distributions

Lithic-rich Initial GSD

Lithic-poor Initial GSD

Weight Fraction

Grain Size (mm)

Spirit Lake

Andreas: Supplemental Figure 2