Mathematical Formulation

We employ an elastic-plastic von Mises yield function (Regenauer-Lieb and Yuen, 2003) together with a power law creep function such that the contributions to the total strain rate, $\dot{\varepsilon}_y^{\text{Total}}$, from the elastic, $\dot{\varepsilon}_y^{\text{elastic}}$, plastic, $\dot{\varepsilon}_y^{\text{plastic}}$, and creep, $\dot{\varepsilon}_y^{\text{creep}}$, strain rates, are additive:

$$\dot{\varepsilon}_y^{\text{Total}} = \dot{\varepsilon}_y^{\text{elastic}} + \dot{\varepsilon}_y^{\text{plastic}} + \dot{\varepsilon}_y^{\text{creep}}$$

or, in terms of the constitutive relation:

$$\dot{\varepsilon}_y^{\text{Total}} = \left(1 + \nu \frac{D\sigma_y}{E Dt} + \nu \frac{Dp}{E Dt} + \alpha \frac{DT}{Dt} \delta_y \right)^{\text{elastic}} + \left(\dot{\varepsilon}_y^{\text{plastic}} \frac{\sigma_y}{2\tau} \right)^{\text{plastic}} + \left(A \sigma_y J_2^{-1} \exp\left[-\frac{Q}{RT}\right]\right)^{\text{creep}}$$

Here, $E$ is Young’s Modulus, $\nu$ is Poisson's Ratio and $\alpha$ is the coefficient of thermal expansion. The operator $\frac{D}{Dt}$ denotes the material derivative. $\tilde{\sigma}_y$ is the objective co-rotational stress rate and $\delta_y$ is the Kronecker delta. $A$ and $n$ are power law material constants, $Q$ is the activation enthalpy, $R$ is the gas constant and $T$ is the absolute temperature. $J_2$ (defined in the caption to Figure 1 in the main text) is the second invariant of the deviatoric stress tensor which in turn is defined as:

$$\sigma_y' = \sigma_y + p\delta_y$$

where $p = -\frac{1}{3}\text{trace}(\sigma_y)$ is the trace of the Cauchy stress tensor, $\sigma_{ij}$, or the pressure. $\tau$ is the plastic yield stress. Constitutive parameters are given in Table 1 in the main text. Notice that there is a weak dependence of the yield function upon the hydrostatic stress. Hence the yield function is better expressed as a Drucker-Prager yield function rather than a von Mises function. However the yield remains associative with no
corners on the yield surface so that localization does not occur without a weakening effect. In the situation explored here, such a weakening effect is supplied by thermal-mechanical coupling provided the system exists in an environment where the ambient temperature is greater than $T_c$ defined in the main text.

The governing equations then are:

**Continuity Equation:** \[ \frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{u} = 0 \]

where $\rho$ is the density and $\mathbf{u}$ is the local material velocity vector. The Continuity Equation incorporates time as a derivative and so is coupled to the Energy Equation below. The Momentum Equation describes equilibrium of forces:

**Momentum Equation:** \[ \nabla \cdot \mathbf{\sigma} + \mathbf{f} = 0 \]

where $\nabla \cdot \mathbf{\sigma}$ is the divergence of the Cauchy stress tensor and $\mathbf{f}$ is the body force.

The Energy Equation describes the energy fluxes during deformation:

**Energy Equation:** \[ \rho c_p \frac{DT}{Dt} = \chi \dot{\varepsilon}_{ij}^{\text{dissipation}} + \chi \frac{Dp}{Dt} + \rho c_p \kappa \nabla^2 T \]

where $c_p$ is the specific heat. The first term on the right of the Energy Equation describes shear heating arising from mechanical dissipative processes where

$\dot{\varepsilon}_{ij}^{\text{dissipation}} = \dot{\varepsilon}_{ij}^{\text{plastic}} + \dot{\varepsilon}_{ij}^{\text{creep}}$. $\chi \leq 1$ is the efficiency of converting mechanical work into heat and we take $\chi = 1$. The second term on the right of the Energy Equation describes the temperature changes arising from isentropic work. The third term describes the temperature changes arising from thermal conduction with thermal diffusivity, $\kappa$.

Note that these three equations are coupled, (i) through the time evolution expressed by the Energy Equation feeding back into the time evolution expressed by the Continuity Equation and, (ii) by the thermal expansion term in the Constitutive Relation which has a feedback influence upon the gradients of stress in the
Momentum Equation. A numerical simulation scheme is clearly needed to follow the evolution of stress and energy with time and this is the topic of this paper.