Strictly speaking, Equation 6 in Kump et al. (2005) is inappropriate for characterizing regional euxinia created by intense local upwelling, as the problem becomes 2- or 3-dimensional. However, the following analysis demonstrates that the inclusion of lateral transport is unimportant for this problem.

To include the non-local sink for H$_2$S requires an additional term in equation 6 that accounts for horizontal eddy diffusion ($D_h$, m$^2$·yr$^{-1}$) (cf. the treatment of worm burrow irrigation by Boudreau, 1997). $D_h$ is scale-dependent (Okubo, 1971); here we have chosen $L=1000$ km, which gives $D_h \approx 3 \times 10^{10}$ m$^2$·yr. We can then express horizontal diffusion in terms of a horizontal piston velocity $\left( \frac{D_h}{L} \right)$. If the flux is radially symmetric, then the horizontal flux of H$_2$S from the upwelling region is through a cross-sectional area of $2\pi LZ$, where $Z$ is the surface layer thickness, and the vertical upwelling flux of H$_2$S from below and the in-mixing flux of O$_2$ from above are through a cross-sectional area $\pi L^2$. Thus, we multiply the maximum horizontal diffusive flux $\left( \frac{D_h}{L} \cdot \rho_{oce} \cdot [H_2S]_{deep} \right)$, when $[H_2S]_{surf} = [H_2S]_{deep}$, by the ratio of these areas $\left( \frac{Z}{L} \right)$ and insert this flux into equation 6. This leads to the following revised steady-state expression for the critical ratio above which the surface layer becomes euxinic given the conditions applied to equation 6:

$$\left( \frac{[H_2S]_{deep}}{P_{O_2, atm}} \right)_{crit} = \frac{k \cdot K_{st}}{2 \cdot \left( u - 2 \cdot \frac{D_h}{L} \cdot \frac{Z}{L} \right)}$$

$$= \frac{1000(\text{m} \cdot \text{yr}^{-1}) \cdot 10^{-3} \text{ mol} \cdot \text{kg}^{-1} \cdot \text{bar}^{-1}}{2 \cdot \left( 100 \text{ m} \cdot \text{yr}^{-1} - 2 \cdot \frac{3 \times 10^{10} \text{ m}^2 \cdot \text{yr}^{-1}}{10^6 \text{ m}} \right) \frac{10^3 \text{ m}}{10^6 \text{ m}}} = 0.005 \text{ mol kg}^{-1} \cdot \text{bar}^{-1}$$

For an atmospheric pO$_2$ of 0.21 bar, the critical $[H_2S]_{deep}$ is 1 mmol·kg$^{-1}$, virtually identical to that obtained by using equation 6 with the appropriate upwelling velocity. In other words, lateral transport is not an important consideration for this problem.

References